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Stabilization vs. Redistribution: The Optimal Monetary-Fiscal Mix[I]

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Abstract

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JEL Classification Numbers: D91, E21, E62.

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1 Introduction

In the public discourse on macroeconomic policy monetary and fiscal policy are typically described as two different entities. With few important exceptions discussed below, this holds for the academic debate as well. For instance, the standard representative-agent New Keynesian framework (henceforth RANK, Woodford 2003, Galí 2008) assumes a stark separation between monetary and fiscal policy.[1] In this paper we show that, in a setting with heterogenous agents and imperfect insurance, monetary and fiscal policy are instead inextricably linked in their optimal design for stabilization and redistribution purposes.

The core argument of our analysis is that, with imperfect insurance, movements in inequality - induced for instance by transfers across agents - are inherently inefficient. A transfer to financially constrained agents reduces inequality, and viceversa. However those variations in inequality generate an inefficiency even under flexible prices, i.e., even in the absence of a nominal rigidity distortion. In order to offset fluctuations in inequality so as to attempt restoring perfect insurance requires using the central bank's leverage over real activity through inflation variations. There is therefore a inherent tension between stabilizing real activity so as to minimize supply-side, price-stickiness distortions and demand-side, imperfect-insurance distortions.

The tax-financing of government spending is a natural setting in which a distributional channel is activated. When government spending is financed by taxation with an asymmetric incidence on the distribution of income (i.e., either progressive or regressive taxation) it creates inefficient fluctuations in income inequality (away from the first-best benchmark of full insurance) even in the absence of a nominal rigidity distortion. To understand this point, suppose that government spending evolves exogenously and is financed with taxes that fall disproportionately more on the share of the population that is unconstrained (i.e., *progressive* taxation). In that case a rise in government spending generates an implicit transfer to constrained agents, who have a higher marginal propensity to consume. The result is a fall in inequality but in fact an inefficient (i.e., too high) redistribution of income from unconstrained to constrained agents, and a *too large* aggregate demand expansion, which is tackled by the policymaker via a monetary contraction (aka a deflation). A symmetric result applies in the case in which higher government spending is financed via *regressive* taxation. In that case, inequality inefficiently rises and the policy authority needs to engineer the optimal degree of monetary accommodation, i.e., to strike a balance between higher inflation and a negative output gap.

In other words, and *because* they give rise to inefficient fluctuations in inequality, exogenous variations in government spending generate a tradeoff between insurance—i.e., minimizing the fluctuations in inequality—and the typical goal of stabilizing aggregate demand and inflation. To be clear, and in the well-known New Keynesian jargon, government spending disturbances act either as "cost-pull" or "cost-push" depending on the degree of progressivity/regressivity in the tax system. In these cases, unless a symmetric distribution of taxes exactly preserves full insurance, the policymaker cannot simultaneously stabilize inflation, the output gap and inequality at their respective efficient levels.

We then study the general monetary-fiscal policy mix when government spending can be *optimally* chosen by the policy authority. We envision a setup where the economy is hit by exogenous "cost-push" shocks, i.e., shocks that generate a time-varying wedge between the efficient level of activity and the one that would prevail under flexible prices. The optimal policy mix in that case calls for a combination of fluctuations in inequality, inflation and real activity away from their respective efficient benchmarks. We show that government spending is used actively, together with monetary policy, to stabilize the economy against those shocks. Interestingly, government spending can be used *pro-* or *counter-cyclically* depending on whether the tax incidence is progressive or regressive. If tax incidence is progressive, a shock that otherwise generates a combination of recession and inflation calls for an expansion in government spending, because the ensuing transfer from unconstrained to constrained agents is the most effective way to stabilize aggregate demand. However, if the incidence of taxes is regressive, optimal policy calls for a *contraction* in government spending even in the face of a recessionary "cost-push" shock. The intuition is that lower public spending financed with regressive taxation generates an implicit transfer towards the constrained agents, thereby effectively boosting aggregate consumption and contributing to stabilizing aggregate demand.

More generally we provide a full analytical characterization of optimal monetary-fiscal policy in an environment with heterogeneous agents and imperfect insurance. We show that the policy mix can call for combinations of monetary accommodation/contraction and fiscal pro- or counter-cyclicality depending on two main factors: the degree of progressivity/regressivity of the tax system and the cyclical behavior of individual income relative to aggregate income.

Throughout the paper, we abstract from public debt; for instance, when we consider transfers, we focus on direct transfers to hand-to-mouth agents financed by taxes on savers (a revenue-neutral, intra-temporal redistribution), rather than uniform lump-sum transfers to all agents financed by issuing debt held by savers, which we interpret as intertemporal redistribution. In our earlier paper, Bilbiie et al (2013), we studied explicitly the role of public debt in relation to redistribution by comparing these policies. One conclusion is that the setup with public debt is isomorphic (in a sense

[1] In the standard RANK setup fiscal policy is typically assumed to be passive (Leeper, 1991), or "Ricardian" (Woodford, 1996) i.e., to behave so as to ensure that the intertemporal government budget equation dictating the dynamics of public debt holds for any price level, or is properly speaking a "constraint"; see also Cochrane (2005).

we make more precise below) to one in which redistribution takes place *intra*-temporally, precisely in the form we study in the present paper.

Related literature In the standard representative-agent NK model (RANK), the benchmark is one of separation of fiscal and monetary policy, important considerations pertaining to "fiscal theory" notwithstanding, see e.g. Leeper (1991), Woodford (1996), Sims (2000) or Cochrane (2005). In the realm of *optimal monetary policy*, the seminal contributions of Clarida, Galí, and Gertler (1999) and Woodford (2003) emphasize that there is no tradeoff between stabilizing inflation and real activity for shocks such as government spending (as well as preference and productivity). A tradeoff opens up instead for so-called "cost-push shocks" making the flexible-price level of output inefficient. Benigno Woodford (2005) showed that when there is a large steady-state distortion (monopolistic competition makes the long-run level of output inefficiently low), government spending does act as a cost-push shock and generates a tradeoff.

In the realm of *fiscal policy*, in the same standard RANK model there is no role for stabilization in normal times. The important exception is when the zero lower bound binds, in a liquidity trap: without the monetary option, governments do optimally resort to fiscal stabilization, as emphasized by Eggertsson (2010), Woodford (2011), Nakata (2016), Schmidt (2013), or Bilbiie, Monacelli and Perotti (2019).

Household heterogeneity and liquidity constraints are key in the two-agent literature (TANK) for *fiscal multipliers* of spending, as for instance in Galí, Lopez-Salido and Vallés (2007), Bilbiie and Straub (2004), and Monacelli and Perotti (2012); for *transfers*, see Bilbiie, Monacelli and Perotti (2013), Mehrotra (2017), Giambattista and Pennings (2017); and for both government spending and transfers in a liquidity trap, see Eggertsson and Krugman (2012). Part of the burgeoning "HANK" literature also studies fiscal multipliers and how they are shaped by market incompleteness and various dimensions of heterogeneity, see Ferrière and Navarro (2023), Auclert, Rognlie, and Straub (2023), Hagedorn, Manovskii, and Mitman (2023) for quantitative models, and Bilbiie (2018, 2020) and Cantore and Freund (2020) for tractable HANK models.

Most importantly, this paper is related to the literature studying optimal policy with heterogeneity and sticky prices. One stream to which this paper belongs firmly uses tractable HANK or TANK models and emphasizes an "inequality motive" (Bilbiie 2008, 2018) associated to cyclical inequality, or the unequal incidence of aggregate fluctuations on different agents—inherently related to profits or financial income accruing to only a subset of agents. That is also of the essence here, as it is in subsequent rich-heterogeneity extensions reviewed below. Using a tractable HANK model with liquidity (money) to solve for Ramsey-optimal policy, Bilbiie and Ragot (2020) emphasize the interaction of an inequality motive with liquidity provision for insurance purposes. Other studies focus on the implications of idiosyncratic risk in a zero-liquidity economy (Challe, 2019) or on quantitative easing (Cui and Sterk, 2019).

A different important literature that we view as complementary to our approach studies optimal policy in rich-heterogeneity, quantitative HANK models, in particular the seminal contribution of Bhandari, Evans, Golosov, and Sargent (2021) which encompasses some of the channels we distill analytically, among others.[2] In a closely-related, subsequent complementary approach, Acharya, Challe, and Dogra (2023) also start with a rich-heterogeneity HANK model with idiosyncratic risk an simplify heterogeneity by building on the CARA-normal structure used in Acharya and Dogra (2020) to obtain a second-order approximation of the aggregate welfare function and analytical insights that complement ours.

Since our analysis deals with the fiscal-monetary policy mix and its optimal design, it is perhaps most closely related to rich-heterogeneity HANK models with a similar focus. In that respect, our "equivalence result" presented below (between transfers and real interest rates as aggregate demand tools) has a general, rich-heterogeneity equivalent shown in Wolf (2021). McKay and Wolf (2023) similarly analyze the optimal design of the monetary-fiscal policy mix but focus instead on an environment with rich heterogeneity in which they derive optimal policy rules; their findings are thus also complementary to ours.[3] Legrand, Martin-Baillon, and Ragot (2023) also study monetary-fiscal stabilization with a large menu of distortionary taxes and liquidity; among other findings, they also show a general version of our "separation" result in their environment with richer heterogeneity (McKay and Wolf prove a related result in their economy, with transfers only).

Our contribution in a nutshell is the *joint* study of optimal monetary and fiscal policies, and the *analytical* approach to the problem—that we transform into a linear-quadratic problem. The latter framework allows us to isolate the novel channel of interaction through fiscal redistribution that affects the stabilization power and motives of the two policies, and thus generates a meaningful tradeoff.

[2] See also Nuno and Thomas (2022) for an early contribution on Ramsey policy in quantitative HANK models focusing on the redistributive role of inflation.

[3] See also Davila and Schaab (2023) for a related rich-heterogeneity study of optimal policy.

2 Stabilization and Redistribution in a Heterogenous Agent Economy

We outline below a simple two-agent New Keynesian model and employ it to characterize analytically the optimal monetary-fiscal mix. The insights of this model transcend this particular setup—see Bilbiie (2018, 2020) and Debortoli and Galí (2024) for formal comparisons of a variety of TANK and HANK models. We view this model as the simplest way to capture inequality and redistribution over the business cycle, as well as the unequal incidence of taxes used to finance government spending across agents depending on their income and wealth.

2.1 A Simple Two-Agent Model

The economy is populated by a continuum of households of measure 1 and by a continuum of firms each producing a differentiated good.

Households A mass λ are "hand to mouth" (labeled by H): they do not participate in the financial markets, but earn labor income and receive fiscal transfers. A mass $1-\lambda$ of households (labeled by S) are permanent-income agents, saving and trading in one-period bonds. They also receive labor income and profits from holding the shares of monopolistic competitive firms. Financial markets are assumed to be complete *within* each group of households, but not *across* groups. All households consume an aggregate basket of differentiated goods, indexed by $i \in [0,1]$, $C_t^{(\theta-1)/\theta} = \int_0^1 C_t\,(i)^{(\theta-1)/\theta}\,di$ where $\theta > 1$ is the constant elasticity of substitution across each pair of varieties.

A generic agent of type $j = H, S$ seeks to maximize intertemporal utility given by $\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t U(C_t^j, N_t^j, G_t^j)\right\}$, where $U(C_t^j, N_t^j, G_t^j)$ is additively separable in private consumption C_t^j , hours worked N_t^j, and consumption of public goods G_t^j.

We assume that in the absence of frictions, government spending is set according to a typical "Samuelson condition" for efficient public good provision, $U_C^j\,(C) = U_G^j\,(G)$. This condition states that the marginal utilities of private and public expenditure should be equalized. We study how this prescription changes when introducing relevant frictions (while maintaining it in the *steady state* of our economy).

The period-by-period budget constraint of agent H (expressed in units of aggregate consumption) reads:

$$C_t^H = W_t N_t^H - T_t^H + \frac{\tau^D}{\lambda}\mathcal{D}_t \tag{1}$$

where $P_t^{1-\theta} = \int_0^1 P_t\,(i)^{1-\theta}\,di$ is the utility-based aggregate price index, W_t the real wage, T_t^H real lump-sum taxes, and $\frac{\tau^D}{\lambda}\mathcal{D}_t$ transfers of profits $\mathcal{D}_t$ received from monopolistic firms (owned by S agents) and which are taxed at the rate τ^D. The latter captures endogenous redistribution through fiscal policy in our model in a transparent way.

The budget constraint of agent S reads:

$$C_t^S + P_t^{-1}B_t^S = P_t^{-1}B_{t-1}^S(1+i_{t-1}) + W_t N_t^S - T_t^S + \frac{1-\tau^D}{1-\lambda}\mathcal{D}_t \tag{2}$$

where B_t^S denotes the holdings of one-period nominal discount bonds (with net nominal interest rate i_t). Notice that in the case $\tau^D = 0$ all profits accrue to S agents, whereas $\tau^D > 0$ implies a redistribution from S to H.

Aggregate consumption is:

$$C_t = \lambda C_t^H + (1-\lambda)C_t^S.$$

The labor supply choice of each agent yields the standard condition:

$$-U_N^j(N_t^j) = W_t U_C^j(C_t^j),\ j = H, S. \tag{3}$$

Finally, only savers participate in asset markets and demand bonds according to the Euler equation:

$$U_C^S(C_t^S) = \beta\mathbb{E}_t\left[\frac{1+i_t}{\Pi_{t+1}}U_C^S(C_{t+1}^S)\right], \tag{4}$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the gross rate of inflation.

Firms There is a continuum of firms, each producing a differentiated good $i \in [0,1]$ with the linear technology: $Y_t(i) = N_t(i)$. The real marginal cost of production, common across firms, is W_t. Each firm faces quadratic price-adjustment costs given by the expression $\Psi\,(\cdot) = \frac{\psi}{2}\left(P_t(i)/P_{t-1}(i) - 1\right)^2 P_t Y_t$, and maximizes intertemporal discounted profits:

$$\max_{P_{i,t}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \Lambda_{0,t}^{s} \left[(1+\tau^{s}) P_t(i) Y_t(i) - W_t N_t(i) - \Psi(\cdot) - T_t^F \right] \right\}$$

subject to a sequence of demand constraints $Y_t(i) = (P_t(i)/P_t)^{-\theta} Y_t$, where $\Lambda_{0,t}^S$ is agent S's stochastic discount factor, τ^s is a sales subsidy financed with a lump-sum tax $T_t^F = \tau^s Y_t(i)$. This ensures that aggregate profits are equal to $\mathcal{D}_t = Y_t - W_t N_t - \Psi(\cdot)$.[4]

In a symmetric equilibrium with $P_t(i) = P_t$ and $Y_t(i) = Y_t$ the above problem leads to the standard non-linear Phillips curve:

$$\Pi_t(\Pi_t - 1) = \mathbb{E}_t \left[\Lambda_{t,t+1}^{S} \frac{Y_{t+1}}{Y_t} \Pi_{t+1}(\Pi_{t+1} - 1) \right] + \frac{\theta}{\psi} \left(W_t - (1+\tau^s) \frac{\theta - 1}{\theta} \right). \tag{5}$$

Market clearing Equilibrium in the goods market requires:[5]

$$Y_t = C_t + G_t + \Delta_t$$

where C_t and G_t correspond to aggregate private and public demand, and $\Delta_t \equiv (\psi/2) Y_t (\Pi_t - 1)^2$ is the portion of aggregate demand absorbed by the price adjustment (where we imposed symmetry). In a symmetric equilibrium, we can thus finally write: $Y_t \left[1 - \frac{\psi}{2} (\Pi_t - 1)^2\right] = \lambda C_t^H + (1-\lambda) C_t^S + G_t$.

Optimal subsidy and steady state In the steady state the real marginal cost (wage) is $W = (1+\tau^s)\frac{\theta-1}{\theta}$. We assume that the subsidy τ^S is set optimally to $(\theta-1)^{-1}$, so that $W = 1$: the steady state is "undistorted" in Woodford's (2003) terminology. Furthermore, this optimal subsidy implies that, when steady-state hours are equalized across agents as we assume $N^H = N^S = N$, steady-state consumption is *also* equalized across agents, effectively implying *full consumption insurance* $C^H = C^S = C$.

3 Inequality and monetary policy tradeoffs

The key point of our analysis is that, via (consumption) inequality and redistribution, the conduct of monetary and fiscal policy are inextricably intertwined. This feature delivers a novel tradeoff for macroeconomic policy. On the one hand, stabilization around the flexible-price level of output triggers distributional inefficiencies; on the other, eliminating imperfect-insurance distortions through redistribution induces inefficient movements in inflation, due to the presence of nominal rigidities.

3.1 Pure transfers

We start by illustrating the tradeoff between stabilization and redistribution in the simplest fiscal policy setting, featuring purely redistributive transfers from savers to the hand-to-mouth.[6] We assume that *government spending is zero*:

$$G_t = T_t = 0$$

Therefore:

$$\lambda T_{H,t} = -(1-\lambda) T_{S,t}$$

Fiscal policy then merely consists of *transfers* across agents. We assume that H receive a transfer that is zero in steady state. Denote $t_{H,t} \approx \left(T_t^H - 0\right)/Y$ and define:

$$t_{H,t} \equiv -f_t, \tag{6}$$

with $t_t^S = \frac{\lambda}{1-\lambda} f_t$ capturing *exogenous* redistribution, to be contrasted with the endogenous redistribution (through τ^D/λ) emphasized previously.

Let $\sigma^{-1} \equiv U_{CC} C / U_C$ and $\varphi \equiv U_{NN} N / U_N$, and lower case letters denote percentage deviations from respective steady state values. Log-linearizing (3) for both agents around a steady state with zero inflation and with the optimal subsidy scheme $\tau^S = (\theta-1)^{-1}$ in place we can write the real wage:

[4] The subsidy scheme is a normalization which does not affect the generality of our analysis and results.

[5] As usual, this holds for each variety and also, because the equilibrium is symmetric, at the aggregate level.

[6] We refer the interested reader to Bilbiie et al (2013) for a study of the *positive* implications of public debt in relation to redistribution in a TANK economy. There, we show that public debt matters for aggregate dynamics only insofar as it changes inflation and the real interest rate. See also Bhattarai et al (2023) for a recent study of the role of debt and inflation in a TANK economy.

$$w_t = \left(\sigma^{-1} + \varphi\right) c_t \tag{7}$$

Log-linearizing the H budget constraint we obtain

$$c_t^H = \left(1 - \frac{\tau^D}{\lambda}\right) w_t + n_t^H - t_{H,t} \tag{8}$$

Combining with the H version of (3) we can write H's consumption as a function of aggregate consumption and redistribution via transfers:

$$\begin{aligned} c_t^H &= \chi c_t - z t_{H,t} \\ &= \underbrace{\chi c_t}_{\substack{\text{sensitivity} \\ \text{to aggregate} \\ \text{income}}} + \underbrace{z f_t}_{\substack{\text{exogenous} \\ \text{redistribution}}} \end{aligned} \tag{9}$$

where

$$\chi \equiv 1 + \varphi\left(1 - \frac{\tau^D}{\lambda}\right); \; z \equiv \frac{1}{1 + (\varphi\sigma)^{-1}}$$

Notice that χ is a parameter (defined as in Bilbiie 2008, 2018) that captures the *elasticity of hand-to-mouth income* to aggregate income. This is a function of the profit redistribution scheme (at the rate τ^D) from S to H. The second term in (9) summarizes the impact of fiscal variables on H. The coefficient z is the elasticity of H consumption to a transfer and governs the strength of the income effect relative to substitution: it is zero when labor supply is infinitely elastic $\varphi = 0$, and it is 1 (largest) when labor supply is inelastic, or when the income effect σ^{-1} is nil.

Consumption of S can in turn be written:

$$c_t^S = \frac{1 - \lambda\chi}{1 - \lambda} c_t - \frac{\lambda}{1 - \lambda} z f_t \tag{10}$$

Inequality

We define *inequality* in consumption or post-tax-and-transfer, net income, as:

$$\gamma_t \equiv c_t^S - c_t^H = \frac{1 - \chi}{1 - \lambda} c_t - \frac{1}{1 - \lambda} z f_t \tag{11}$$

Hence inequality falls (rises) with aggregate income to the extent that $\chi > (<)1$, and it falls with redistribution via transfers ($f_t > 0$).

Aggregate demand and supply Log-linearizing (4) and combining with (10) we can express the *aggregate* consumption Euler equation as:

$$c_t = \mathbb{E}_t c_{t+1} - \sigma \frac{1 - \lambda}{1 - \lambda\chi} \left(i_t - \mathbb{E}_t \pi_{t+1}\right) + \underbrace{\frac{\lambda}{1 - \lambda\chi} z \left(f_t - \mathbb{E}_t f_{t+1}\right)}_{\substack{\text{role of} \\ \text{transfers}}} \tag{12}$$

The last term on the right hand side captures the (intertemporal) effect on aggregate consumption of exogenous redistribution via transfers. This aggregate-demand equation echoes Bilbiie, Monacelli and Perotti (2013, eq. 24), and allows us to emphasize in Proposition 1 a key property pertaining to monetary and fiscal policy as aggregate demand management tools in this model.

Proposition 1 ***Monetary-Fiscal Aggregate-Demand Equivalence***. *Real interest rates $r_t \equiv i_t - \mathbb{E}_t \pi_{t+1}$ and fiscal transfers are substitutes for aggregate-demand management: i.e., to achieve the same path of aggregate consumption $\{c_t\}$ policymakers can equivalently employ a path for transfers $\{f_t\}$ or an interest rate path $\{-r_t\}$ as long as:*

$$\sigma (1 - \lambda) (-r_t) = \lambda z \left(f_t - \mathbb{E}_t f_{t+1}\right). \tag{13}$$

The above proposition points to an equivalence between the real interest rate and transfers from the standpoint of aggregate demand management. The effect of either policy is respectively summarized by the terms on the left and right hand side of equation (13). That equation captures the total direct, partial-equilibrium effect of either policy on aggregate demand: intertemporal substitution σ by $(1-\lambda)$ agents, for interest rate cuts; and direct increases in consumption with elasticity z (the income effect of the transfer) for the λ hand-to-mouth agents. Both policies then get amplified in general equilibrium through an indirect effect (reminiscent of a "Keynesian cross") summarized by the term $\lambda\chi$ in equation (12), which acts like an aggregate MPC.[7] This feature is likely to transcend our simple model and extend to richer models in the HANK class insofar as they feature the following mechanisms: a share of high-MPC agents, unequal incidence of aggregate income on individual incomes (captured by χ), and net fiscal transfers between low- and high-MPC agents, including the ones operating indirectly through the progressivity of the tax system. Indeed, in a recent complementary contribution Wolf (2021) shows a more general version of this result in a rich HANK environment.

Log-linearizing the optimal pricing condition (5) around a zero inflation steady state with perfect insurance leads (in a symmetric equilibrium) to a standard New Keynesian Phillips-curve:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa c_t \tag{14}$$

where the slope parameter is given by $\kappa \equiv \frac{\theta}{\psi}\left(\sigma^{-1}+\varphi\right)$.

Equilibrium with pure transfers

For a given specification of monetary and fiscal policy $\{i_t, f_t\}$ a rational expectations equilibrium with transfers and costly price adjustment is a sequence of processes $\{c_t, \pi_t\}$ solving equations (12) and (14).

Flexible price allocation

Equation (7) allows us to solve for the *natural* level of output, i.e., the level of output that occurs under flexible prices (denoted with a star), whereby markups are constant at their desired level and the marginal cost is also constant, $w_t^* = 0$. We therefore have:

$$c_t^* = 0 \tag{15}$$

In turn, inequality under flexible prices reads

$$\gamma_t^* = -\frac{z}{1-\lambda} f_t \tag{16}$$

Notice that a transfer to the constrained H agents reduces inequality, and viceversa. However, and to the extent that insurance is imperfect, those movements in inequality generate an inefficiency even under flexible prices, i.e., even in the absence of a nominal rigidity distortion.

Perfect insurance allocation

The equilibrium under perfect insurance requires instead

$$\gamma_t = 0 \equiv \gamma_t^{**} \text{ for all t}$$

Hence, from (11), we have:

$$c_t^{**} = -\frac{z}{\chi-1} f_t \tag{17}$$

Therefore transfers generate a time-varying gap between the flexible-price c_t^* and the perfect-insurance c_t^{**} level of consumption (output). In the absence of transfers ($f_t = 0$ for all t), the flexible-price and the perfect-insurance level of output coincide and are in turn equal to the efficient level of output.

Assuming (without loss of generality) that f_t follows an iid process allows to derive, from (14), a solution for inflation under perfect insurance:

$$\pi_t^{**} = -\frac{\kappa z}{\chi-1} f_t \tag{18}$$

Equations (17) and (18) show that, under full insurance, a transfer towards the high-MPC hand-to-mouth agents generates an effect on inflation and real activity whose sign depends on the value of χ.

A transfer towards the H agents has *three* distinct effects. To start with, the transfer *decreases inequality* mechanically, through a direct effect: it is an exogenous income windfall for H, corresponding to the second term in (9). Second, however, there is an indirect effect on inequality through the first term in (11). To see this, notice that the transfer towards the H agents is always *expansionary* on aggregate demand (at given real interest rates, i.e., for a given monetary policy stance), regardless of how individual incomes comove with the cycle, i.e., regardless of χ. The ensuing effect on inequality, however, depends on the value of χ. If the income of H over-reacts endogenously to the above increase in

[7] See Bilbiie (2020) for a further elaboration of that point.

aggregate income ($\chi > 1$), inequality then falls further, amplifying the initial direct effect. This movement, as we shall see formally below, is inefficient. To restore perfect insurance, the central bank needs to engineer a recession (and therefore a deflation) that increases the inequality back towards zero and thus closes the distributional gap. Conversely, if income of the hand-to-mouth instead under-reacts to aggregate income, $\chi < 1$, the aggregate expansion leads endogenously to an *increase* in inequality back towards its optimal level (of zero); to close the gap entirely, the central bank needs to act procyclically, so as to further amplify the expansion (and inflate) up to the point where the direct effect through the transfer has been neutralized.

There are thus three relevant channels stemming from transfers: (i) a direct effect on income inequality; (ii) an (indirect) effect of the former on aggregate demand, which depends on the equilibrium monetary policy response; (iii) an indirect effect on income inequality through its endogenous response to aggregate income. Put differently, neutralizing fluctuations in inequality so as to attempt restoring perfect insurance requires using the central bank's leverage over real activity through inflation variations. There is therefore a inherent tension between stabilizing real activity so as to minimize *supply-side*, price-stickiness distortions (via a policy of zero inflation) and *demand-side*, imperfect-insurance distortions. As we will now see, optimal policy consists of balancing this tradeoff.

Welfare objective

To evaluate aggregate welfare when calculating optimal policy, we assume that the policymaker uses a utilitarian welfare criterion. In the Appendix we derive a second-order approximation to intertemporal weighted utility around the non-distorted steady state, obtaining the following proposition.

Proposition 2 *The social welfare function is proportional to the quadratic loss $\mathcal{L}_t$:*

$$\mathcal{L}_t \equiv \frac{1}{2}\left\{\pi_t^2 + \underbrace{\frac{\sigma^{-1}+\varphi}{\psi}c_t^2}_{output\ gap} + \underbrace{\frac{(\sigma z)^{-1}}{\psi}\lambda(1-\lambda)\gamma_t^2}_{inequality}\right\} \tag{19}$$

or alternatively, replacing inequality using (11):

$$\begin{aligned} \mathcal{L}_t &\equiv \frac{1}{2}\left\{\pi_t^2 + \frac{\sigma^{-1}+\varphi}{\psi}c_t^2 + \frac{\sigma^{-1}+\varphi}{\psi}\Theta\left(c_t + \frac{z}{\chi-1}f_t\right)^2\right\}, \\ \text{where } \Theta &\equiv \frac{\lambda(\chi-1)^2}{(1-\lambda)\varphi\sigma}. \end{aligned} \tag{20}$$

ignoring terms of order higher than 2 and terms independent of policy.

Notice that in the case $\lambda = 0$, (19) reduces to the quadratic loss function typically derived in RANK models (e.g., Woodford, 2003) that depends on two components. The first component captures the real output loss stemming from inflation movements, due to the presence of costly price adjustment. The second component captures the welfare loss stemming from aggregate output deviating from its *flexible-price* level. In our framework, however, the welfare loss features a novel term stemming from inefficient movements in inequality that are generated by fiscal *transfers***.**

The composite parameter Θ captures the effect of the additional inequality-stabilization motive. The presence of an inequality component in the welfare loss is not novel *per se*: it is a typical feature of a two-agent NK model (Bilbiie 2008, 2018). What *is* novel in our case is that those losses stem from transfers generating variations in inequality, even conditional on a zero output gap (as equation (11) shows). In other words, the inequality term is *not* proportional to the output gap relative to flexible prices—there is a *wedge* between the two, driven by redistribution. Put differently, the presence of transfers across agents generates an endogenous wedge (proportional to inequality) between the welfare-relevant and the flexible-price level of output. Hence movements in inequality induced by fiscal transfers make the flexible-price equilibrium suboptimal.

Optimal transfers In the absence of other shocks, since transfers induce suboptimal variations in inequality their optimal value would be zero. In designing the optimal policy problem we therefore distinguish two cases. In the first case, transfers are exogenous. In the second case, they can be chosen optimally in the presence of other disturbances that generate a meaningful tradeoff for policy. We analyze those cases next.

Optimal policy with exogenous transfers

Throughout, and without loss of generality, we assume that the policy authority acts under discretion, i.e., it re-optimizes period-by-period taking private agents' future expectations as given.; results under (timeless-optimal) commitment are similar, i.e., they differ from the RANK benchmark in the same way that results under discretion do. If fiscal transfers are treated as exogenous the optimal policy problem under discretion consists of minimizing the loss function (20) subject to the Phillips curve constraint (14).

The first order condition reads:

$$\kappa\pi_t + \frac{\sigma^{-1}+\varphi}{\psi}c_t + \frac{\sigma^{-1}+\varphi}{\psi}\Theta\left(c_t + \frac{z}{\chi-1}f_t\right) = 0 \tag{21}$$

Notice that non-zero exogenous fiscal transfers make it suboptimal to fully stabilize inflation and the output gap: in other words, exogenous redistribution introduces a stabilization trade-off. In the limit case of a RANK economy ($\lambda = 0 \to \Theta = 0$), condition (21) reproduces the standard optimality prescription in the absence of any tradeoff for the policy authority, $\pi_t = c_t = 0$.[8]

We can derive the equilibrium solution for c_t, π_t, as well as inequality γ_t under the optimal policy by combining (14) and (21) and assuming that f_t follows a stationary process with persistence $p \in [0,1]$:

$$\begin{aligned} c_t &= -\frac{\lambda}{1-\lambda}\frac{\chi-1}{1+\sigma\varphi}\Phi f_t, \\ \pi_t &= -\frac{\lambda}{1-\lambda}\frac{\chi-1}{1+\sigma\varphi}\frac{\kappa}{1-\beta p}\Phi f_t, \\ \gamma_t &= -\frac{z}{1-\lambda}\left(1+\frac{\kappa\theta}{1-\beta p}\right)\Phi f_t, \end{aligned} \tag{22}$$

where we defined the composite parameter $\Phi \equiv \left[1+\frac{\kappa\theta}{1-\beta p}+\Theta\right]^{-1} > 0$.

Hence a transfer between agents requires an optimal policy response which depends on whether the endogenous redistribution of aggregate income makes inequality countercyclical ($\chi > 1$) or procyclical ($\chi < 1$).

Let's consider the case of a transfer towards the H agents ($f_t > 0$). If endogenous redistribution is such that the income of hand-to-mouth over-reacts to the aggregate ($\chi > 1$) optimal policy, (22) requires the policymaker to engineer a contraction in economic activity and a deflation along with a decrease in income inequality. The intuition lies in our previous discussion. *Ceteris paribus* the transfer generates an expansion, but that expansion is suboptimal (the optimal transfer is zero), hence optimal policy needs to act so as to compensate for that by generating an aggregate recession. If the income of H over-reacts endogenously to the transfer ($\chi > 1$), the downward effect on inequality is amplified relative to the initial direct effect. To restore perfect insurance, the policy authority optimally engineers a recession (and a deflation) that increases inequality back towards zero and thus partly closes the distributional gap. Conversely, if $\chi < 1$ a fiscal transfer generates an *increase* in inequality back towards its optimal level (of zero). To close the distributional gap, in that case, the central bank needs to act procyclically, so as to further amplify the expansion (and inflate) up to the point where the direct effect through the transfer has been neutralized.

A further useful interpretation is obtained by looking at the response of the output *gap* relative to the perfect-insurance level c_t^{**} given by (17), namely:

$$c_t - c_t^{**} = \frac{z}{\chi-1}\left(1+\frac{\kappa\theta}{1-\beta p}\right)\Phi f_t.$$

Thus, in the case $\chi > 1$, when transfers to H are deflationary under optimal policy, they are expansionary relative to the perfect-insurance equilibrium (the output gap relative to c_t^{**} is positive, even though it is negative relative to the flexible-price level). Thus, exogenous transfers generate a negative comovement under optimal policy between inflation and a welfare-relevant, redefined concept of "output gap": in New Keynesian jargon, they act as "cost-pull" shocks when $\chi > 1$ and "cost-push" when $\chi < 1$.

[8] This is the special case typically labelled, in the New Keynesian literature, as "divine coincidence".

Optimal policy with endogenous transfers: a separation proposition

Next we study the case in which fiscal transfers are chosen optimally.[9] To that goal we assume the existence of exogenous ("cost-push") shocks u_t as additive terms in the Phillips-curve equation. To be specific, consider exogenous shocks to desired-markups driven by changes in the elasticity of substitution between goods $\mu_t^* \equiv \frac{1}{\theta} \ln \frac{\mathcal{M}_t^*}{\mathcal{M}} = -\frac{1}{\theta-1} \frac{\theta_t - \theta}{\theta}$, which imply the loglinearized Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{\theta}{\psi} mc_t + \frac{\theta}{\psi} \mu_t^*. \tag{23}$$

Replacing the expression for marginal cost as a function of aggregate activity (and the definition of κ above) and using the standard cost-push shock notation $u_t \equiv \frac{\theta}{\psi} \mu_t^*$, this becomes:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa c_t + \underbrace{u_t}_{\substack{\text{cost-push}\\ \text{shock}}} \tag{24}$$

The optimal policy problem under discretion consists in minimizing (20) subject to (24) taking the process for u_t and inflation expectations as given.

In addition to (21) we now also have an optimality condition on f_t. Differentiation of (20) with respect to f_t using (21) delivers directly

$$c_t + \frac{z}{\chi - 1} f_t = 0 \tag{25}$$

This implies Proposition (3).

Proposition 3 *When both policies are chosen optimally, a separation principle applies: monetary policy stabilizes the cost-push shock, while the fiscal transfer is chosen so as to achieve perfect insurance. The resulting equilibrium is thus identical to the optimal-policy allocation of the representative-agent model.*

At a broad level, this proposition is just an implication of the classic Tinbergen-Theil principle: there are as many instruments as targets. Assuming that u_t follows a stationary process with persistence $p \in [0, 1]$ the Phillips curve equation (24) reads:

$$(1 - \beta p) \pi_t = \kappa c_t + u_t \tag{26}$$

The system (21), (25), and (26) then delivers the same allocation as the RANK model (Clarida et al 1999, Woodford 2003):

$$c_t = -\frac{\theta}{\theta\kappa + 1 - \beta p} u_t; \; \pi_t = \frac{1}{\theta\kappa + 1 - \beta p} u_t.$$

In addition, the optimal transfer that delivers perfect redistribution over the cycle is:

$$f_t = \frac{\chi - 1}{z} \frac{\theta}{\theta\kappa + 1 - \beta p} u_t.$$

Hence in the face of exogenous inflationary pressures the policymaker optimally employes transfers (i.e., redistribution) to stabilize the economy, depending on the sign of $\chi - 1$. As in a standard RANK economy ($\lambda = 0$) a positive cost-push shock requires an optimal balance between positive inflation and negative output gap. In the TANK economy, however, the contraction in activity also implies an effect on inequality, which in turn depends on the value of χ. In the case of $\chi > 1$ the fall in real activity exerts an upward pressure on income inequality, and therefore requires a transfer towards the constrained agents (a rise in f_t) to stabilize inequality. The reverse holds in the case $\chi < 1$, with an inflationary shock requiring a transfer away from the H agents.[10]

3.2 Constrained fiscal policy: redistributive effects of cost-push shocks

The previous result applies when both policies are chosen optimally. It is informative to study the case where fiscal policy is instead constrained (assume without loss of generality that $f_t = 0$), which allows us to clarify the redistributional dynamics engendered by cost-push shocks under optimal monetary policy.

[9] Notice that the optimal-policy problem is *block-recursive* in the following sense. The only thing that matters for the optimal-policy problem is "transfers to H", i.e. f_t; having solved for their optimal path, one can pick taxes on S to make sure public debt does not explode and the equilibrium is determinate, but without changing the allocation. Studying the optimal design of public debt therefore requires a richer environment, beyond the scope of this paper.

[10] Some version of this result generalizes to rich-heterogeneity HANK models, as shown subsequently by Legrand et al (2023) in an environment with distortionary taxes and McKay and Wolf (2023) with transfers.

It is important to first note that the loglinearized profits equation does *not* feature the markup (cost-push) shock *independently*. Indeed, under our assumptions of constant returns and around an undistorted steady state, the profit function is simply:

$$d_t = -w_t = -mc_t.$$

Evaluated at the flexible-price equilibrium $\psi = 0$, profits *do* depend on the markup shock since:

$$mc_t^* = -\mu_t^* \text{ and } d_t^* = \mu_t^*$$

Substituting this in the H agents' budget constraint (8) combined with their labor supply schedule, one easily obtains the full equilibrium under flexible prices; in it, aggregate activity and inequality are:

$$\begin{aligned} c_t^* &= n_t^* = y_t^* = -\frac{1}{\sigma^{-1}+\varphi}\mu_t^* \\ \gamma_t^* &= c_t^{S*} - c_t^{H*} = \frac{\chi-1}{1-\lambda}\frac{1}{\sigma^{-1}+\varphi}\mu_t^* \end{aligned}$$

So an increase in desired markups leads to an increase in flex-price profits, an increase in inequality, and a fall in aggregate consumption (the latter of which is the same as it would be in RANK).

The effect of these shocks under sticky prices however depends on the policy response. We saw that under our benchmark whereby both policies are chosen optimally we obtain a separation result: the equilibrium is the same as in RANK. Here, to distill the transmission under optimal policy and the role of profits, we focus on the case where fiscal policy is fixed, and only monetary policy is chosen optimally; this is thus identical to Bilbiie (2008, 2018), but those papers did not solve explicitly for the differential effects on inequality and profits.[11] The first-order condition is thus (21) evaluated at $f_t = 0$; combined with the Phillips curve (24), this delivers equilibrium consumption under optimal monetary policy, given fiscal policy:

$$c_t = -\frac{\theta\Phi}{1-\beta p}u_t = -\frac{\theta}{\kappa\theta + (1-\beta p)(1+\Theta)}\frac{\theta}{\psi}\mu_t^*.$$

As it is well-understood by now, the inequality-stabilization motive implies tolerating a larger inflation response to mitigate the fall in output. We can dissect the intuition for this result further as follows. Notice that the output gap with respect to the flex-price level is positive (output falls by less than under flexible prices):

$$c_t - c_t^* = \frac{1+\Theta}{\frac{\kappa\theta}{1-\beta p} + 1 + \Theta}\frac{1}{\sigma^{-1}+\varphi}\mu_t^*, \tag{27}$$

Turning to the response of profits:

$$d_t = \frac{\kappa\theta}{\kappa\theta + (1-\beta p)(1+\Theta)}\mu_t^*, \tag{28}$$

it is apparent that they respond less to the markup shock than in RANK (or in an equal-incidence $\chi = 1$ TANK economy), because of the additional inequality-stabilization motive modulated by Θ. Relative to their flex-price level:

$$d_t - d_t^* = -\frac{1+\Theta}{\frac{\kappa\theta}{1-\beta p} + 1 + \Theta}\mu_t^*, \tag{29}$$

profits increase *less*, and the gap is increasing in absolute value with heterogeneity Θ.

Finally, consider the response of inequality:

$$\gamma_t = -\frac{1-\chi}{1-\lambda}\frac{1}{\sigma^{-1}+\varphi}\frac{\kappa\theta}{\kappa\theta + (1-\beta p)(1+\Theta)}\mu_t^*, \tag{30}$$

Since we consider $\chi > 1$, inequality generically increases following a markup shock. However, because of the profits' mitigated response, it increases by *less* than under flexible prices, i.e., the "inequality gap" is:

$$\gamma_t - \gamma_t^* = \frac{1-\chi}{1-\lambda}\frac{1}{\sigma^{-1}+\varphi}\frac{1+\Theta}{\frac{\kappa\theta}{1-\beta p} + 1 + \Theta}\mu_t^*. \tag{31}$$

[11] To be entirely correct, Section 7 of the 2004 working paper version of Bilbiie (2008) does solve analytically for the cyclical implications of several demand and supply shocks, including on profits but not (explicitly) on inequality.

Inequality increases less than under flexible prices (because profits increase less) which allows the policymaker to achieve a better tradeoff balancing the standard stabilization motive materializing into an output-inflation tradeoff with the inequality-stabilization motive. These results and intuition complement and echo not only the TANK insights developed originally in Bilbiie (2008, 2018)—see Proposition 4 in the former and Proposition 7 in the latter—but also the HANK results obtained quantitatively by Bhandari et al (2021) and explored analytically by Acharya et al (2023), see their Figure 6 in Section 5.1.

3.3 Optimal monetary policy and inequality: the case of an inequality-distorted steady state

All our results hinge upon the approximation around an "equal" efficient equilibrium, a steady state with equal consumption across agents—so variations in inequality of any sign are costly for aggregate welfare. Next, and for this section *only.* we relax this assumption and study the case of an unequal steady state whereby hand-to-mouth agents have a lower consumption "on average", i.e., in steady state; nevertheless, we focus on the case whereby this difference is "small", perhaps because fiscal policy already achieves a large amount of redistribution in the long run. We extend the basic second-order approximation to welfare to the case of a "distorted" steady state, where "distorted" refers to the cross-sectional, inequality dimension. To be specific, let:

$$\Gamma \equiv \frac{C^S}{C^H} \geq 1 \tag{32}$$

denote stead-state consumption inequality. We consider the case of a "small" distortion, whereby fiscal policy does much of the redistribution in the long run but there is a first-order term left that might affect the monetary authority's job (technically, $\Gamma - 1$ is "of order 1"). We show that this gives rise to a linear term in the quadratic loss function: much as in RANK where a small supply-side, monopolistic distortion imparts an average inflation bias but does not affect the stabilization properties, we show that in TANK this extension has a similar effect.

To isolate this point, we consider (again, for this section only) a simpler economy: we abstract from fiscal transfers used for stabilization purposes (if a transfer were available, it would be used to perfectly close this new first-order relevant welfare gap). Furthermore, for simplicity we consider the case of a unionized labor market often used in models with sticky wages but also in some TANK models, including in the context of optimal policy (e.g. Bilbiie and Ragot, 2020). Concretely, we assume that hours worked are pooled by a union and labor is demand-determined—thus following Galí et al, 2007 and in particular Ascari et al, 2017 for a setup with a union and sticky wages studying optimal policy. The equilibrium implication is that hours worked by each agent are identical:

$$N_t^H = N_t^S = N_t,$$

and determined by an aggregate hours schedule that we specify as:

$$w_t = \xi \left(N_t\right)^{\varphi} \left(C_t\right)^{\sigma^{-1}}.$$

Steady state. Differently from the benchmark case of an efficient steady state (including perfect insurance), we now consider an economy with arbitrary steady-state inequality. We assume that the monopoly distortion is still eliminated by a sales subsidy (this allows us to abstract from a well-understood distortion in New Keynesian models); but this subsidy is now financed by taxing *both* households, in an arbitrary way. Specifically, let η be the fraction of the total subsidy for sales that is financed by taxes on the H group, $T^H = \frac{\eta}{\lambda}\tau^S Y$, with $T^S = \frac{1-\eta}{1-\lambda}\tau^S Y$. We show in the Appendix that the steady state consumption shares are given by

$$\frac{C^H}{C} = 1 + \frac{\tau^D - \eta}{\lambda}\frac{1}{\theta - 1};\ \frac{C^S}{C} = 1 + \frac{\eta - \tau^D}{1-\lambda}\frac{1}{\theta - 1}$$

Therefore, steady state inequality $\Gamma \equiv \frac{C^S}{C^H}$ is modulated by (i) the steady state net markup $\frac{1}{\theta-1}$; (ii) the profit redistribution every period τ^D and (iii) the share of taxes levied on H to finance the subsidy η. Crucially, we assume that this product is "small" so that the distortion is first-order; this amounts to assuming that the hand-to-mouth agents only pay a fraction of the subsidy that is very close to the share of profits they get indirectly through taxation τ^D.

In the Appendix, we prove the following Proposition.

Proposition 4 *The social welfare function is proportional to the linear-quadratic loss $\mathcal{L}_t$:*

$$\mathcal{L} = \frac{1}{2}\left\{\Lambda_c c_t^2 + \pi_t^2\right\} - \Lambda c_t, \tag{33}$$

with

$$\Lambda_c \equiv \frac{\sigma^{-1}+\varphi}{\psi} + \frac{\sigma^{-1}}{\psi}\frac{\lambda\left(\chi_u-1\right)^2}{1-\lambda}$$

$$\Lambda \equiv \lambda\left(\Gamma^{\sigma^{-1}}-1\right)\frac{\chi_u-1}{\psi}$$

ignoring terms of order higher than 2 $O\left(\parallel \zeta \parallel^3\right)$ *and terms independent of policy.*

Denoting consumption inequality (as deviation/gap, in percentage of steady state consumption) by:

$$\hat{\gamma}_t = \frac{C^H}{C}c_t^H - \frac{C^S}{C}c_t^S$$

it is clear that the linear term becomes:

$$\lambda\left(1-\lambda\right)\left(\Gamma^{\sigma^{-1}}-1\right)\hat{\gamma}_t, \tag{34}$$

substantiating our claim that in this economy there is a first-order benefit to providing insurance and reducing inequality, i.e., boosting the consumption of hand-to-moth agents who have a lower consumption level and thus a higher marginal utility of consumption on average. This benefit is intuitively proportional to the degree of the "distortion", i.e. of steady-state inequality.

Evidently, if we reintroduced transfers that could be used over the cycle these would be used to perfectly offset this linear gap. Instead, we consider again the case where fiscal policy is constrained and such transfers are not available, but monetary policy minimizes the loss function subject to the Phillips curve constraint, under discretion.

The targeting rule is:

$$\kappa\pi_t + \Lambda_c c_t - \Lambda = 0 \tag{35}$$

Replacing the Phillips curve with persistence p, equilibrium consumption and inflation are now:

$$\begin{aligned} c_t &= \frac{\Lambda\left(1-\beta p\right)}{\kappa^2+\Lambda_c\left(1-\beta p\right)} - \frac{\kappa}{\kappa^2+\Lambda_c\left(1-\beta p\right)}u_t \\ \pi_t &= \frac{\kappa\Lambda}{\kappa^2+\Lambda_c\left(1-\beta p\right)} + \frac{\Lambda_c}{\kappa^2+\Lambda_c\left(1-\beta p\right)}u_t \end{aligned} \tag{36}$$

As these expressions show clearly, the presence of a small inequality distortion in steady state is very similar to the presence of a monopolistic distortion in the RANK model. Namely, two things are of note.

First, steady state inequality imparts an *average inflation bias*, as the central bank attempts to maintain aggregate activity higher on average when inequality is countercyclical in order to insure hand-to-mouth agents (the same way in which it tries to increase output beyond potential in the RANK model). That incentive is increasing in the share of hand-to-mouth agents, in the degree of steady state inequality Γ, and in the countercyclicality of inequality (in the cyclicality of H agents' income χ_u); intuitively, all these forces increase the incentive of a benevolent policymaker to use its leverage over aggregate activity to "insure" those in hardship.

Second, the *responses to shocks* are *unaffected* by the distorted steady state when the distortion is small. In other words, the response of the economy to shocks under the optimal policy is independent of the small inequality distortion; this result extends to the case with commitment (just as it does in the RANK model with a monopolistic distortion, in that case the deterministic component in inflation also vanishes over time under commitment).

4 Government spending

So far we have assumed that government spending is zero. We show next that the financing of government spending *per se* requires implicit transfers across agents, reproducing similar insights to the ones outlined above in the simplest case of pure transfers.

We go back to the case of an efficient steady state (and decentralized labor market) and assume that the government purchases a basket of the consumption goods G_t with the same composition as the private consumption basket and finances it by lump-sum taxes (balanced-budget) levied on both agents:

$$G_t = T_t = \lambda T_t^H + \left(1-\lambda\right)T_t^S$$

Thus, any expansion in government spending must be accompanied by increasing taxes. In our heterogeneous agent environment, however, this adjustment entails a redistributive content. To capture redistribution across agents we assume that agents H pay an arbitrary share α of total taxes:

$$\lambda T_t^H = \alpha T_t.$$

In turn, agents S pay $(1-\lambda) T_t^S = (1-\alpha) T_t$.

Let's define $t_{H,t} \approx \left(T_t^H - G\right)/Y$ and $g_t \approx \left(G_t - G\right)/Y$). We can then decompose $t_{H,t}$ as:

$$t_{H,t} = \frac{\alpha}{\lambda} t_t = \frac{\alpha}{\lambda} g_t = \underbrace{g_t}_{\text{uniform}} - \underbrace{\left(1 - \frac{\alpha}{\lambda}\right) t_t}_{\text{exogenous redistribution.}} \tag{37}$$

In the above expression the taxes on H agents are the sum of a uniform tax (equal to the spending increase g_t) and a transfer to H whenever $\alpha < \lambda$ (a transfer from H otherwise) capturing *exogenous* redistribution.[12]

Flexible-price allocation with government spending Log-linearizing (3) around a steady state with zero inflation and with the optimal subsidy scheme $\tau^S = (\theta - 1)^{-1}$ in place we can write:

$$w_t = \left(\sigma^{-1} + \tilde{\varphi}\right) c_t + \tilde{\varphi} \tilde{g}_t \tag{38}$$

with $\tilde{g}_t \equiv \frac{1}{1-G_Y} g_t$; $\tilde{\varphi} \equiv \varphi \left(1 - G_Y\right)$; and with G_Y being the steady state share of government spending in output.[13]

From (7) we can solve for the *natural* level of output in the presence of government spending, which now reads:

$$c_t^* = -\tilde{z}\tilde{g}_t, \tag{39}$$

$$\text{where } \tilde{z} \equiv \frac{1}{1 + \left(\tilde{\varphi}\sigma\right)^{-1}}$$

Substituting the wage from (7) and (37) into the H budget constraint we obtain expressions for consumption of both agents (where $\tilde{t}_{j,t} \equiv \frac{1}{1-G_Y} t_{j,t}$):

$$\begin{aligned} c_t^H &= \chi c_t - \tilde{z} t_{H,t} = \chi c_t + \tilde{z} \left[(\chi - 1) \tilde{g}_t + \left(1 - \frac{\alpha}{\lambda}\right) \tilde{t}_t \right] \\ &= \chi c_t + \tilde{z} \left(\chi - \frac{\alpha}{\lambda}\right) \tilde{g}_t, \end{aligned} \tag{40}$$

$$\begin{aligned} c_t^S &= \frac{1 - \lambda\chi}{1 - \lambda} c_t - \frac{\lambda}{1 - \lambda} \tilde{z} \left(\chi \tilde{g}_t - \tilde{t}_{H,t}\right) \\ &= \frac{1 - \lambda\chi}{1 - \lambda} c_t - \frac{\lambda}{1 - \lambda} \tilde{z} \left(\chi - \frac{\alpha}{\lambda}\right) \tilde{g}_t. \end{aligned} \tag{41}$$

Replacing (41) in the log-linearized version of the Euler equation of S (4) (having also replaced the expression for transfers' distribution) we obtain the *aggregate* consumption Euler condition:

$$c_t = \mathbb{E}_t c_{t+1} + \frac{\lambda}{1 - \lambda\chi} \tilde{z} \left(\chi - \frac{\alpha}{\lambda}\right) \left(\tilde{g}_t - \mathbb{E}_t \tilde{g}_{t+1}\right) - \sigma \frac{1 - \lambda}{1 - \lambda\chi} \left(i_t - \mathbb{E}_t \pi_{t+1}\right) \tag{42}$$

In line with Proposition 1 we can now state an alternative equivalence result on the role of interest rate and government spending policies for aggregate demand management.

Proposition 5 ***Equivalence between Interest Rate and Government Spending Policies***. *Real interest rates and government spending are substitutes for aggregate-demand management: i.e., to achieve the same path of aggregate consumption $\{c_t\}$ policymakers can equivalently employ a path of government spending $\{g_t\}$ or an interest rate path $\{-r_t\}$ as long as:*

$$\sigma(1 - \lambda) \left(-r_t\right) = \lambda \tilde{z} \left(\chi - \frac{\alpha}{\lambda}\right) \left(\tilde{g}_t - \mathbb{E}_t \tilde{g}_{t+1}\right) \tag{43}$$

[12] We contrast exogenous redistribution with the "endogenous redistribution" working through the taxation of firms' profits.

[13] Note that g_t is already in Y units.

The intuition is similar to the one for Proposition 1: the above terms control the direct effects of the two respective policies on aggregate demand, i.e. the shift of the planned-expenditure line in a Keynesian cross framework. Notice, however, that in this case the effect of government spending policies depends on the characteristics of the underlying tax system (i.e., on whether α is greater or smaller than λ) and on how that interacts with the degree of cyclicality of income inequality (summarized by χ). The reason is that these two features control the endogenous and exogenous redistribution channels that shape the multiplier of government spending, see section 3.2 of Bilbiie (2020) for a detailed discussion.

Phillips curve with government spending Log-linearization of (5) around a zero-inflation steady state under the optimal subsidy yields:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa (c_t + \tilde{z}\tilde{g}_t) = \beta \mathbb{E}_t \pi_{t+1} + \kappa \underbrace{(c_t - c_t^*)}_{\substack{\text{output} \\ \text{gap}}} \tag{44}$$

where $\kappa \equiv \frac{\theta}{\psi}\left(\sigma^{-1} + \tilde{\varphi}\right)$, and the second equality follows from (15).

Inequality and the output gap Inequality in consumption (or post-tax income) is now given by:

$$\gamma_t \equiv c_t^S - c_t^H = \frac{1-\chi}{1-\lambda} c_t - \frac{1}{1-\lambda} \tilde{z} \left(\chi - \frac{\alpha}{\lambda}\right) \tilde{g}_t, \tag{45}$$

An important observation for our analysis is that the variable γ_t is related to the output (consumption) gap through:

$$\gamma_t = \frac{1-\chi}{1-\lambda}\left(c_t - c_t^*\right) - \frac{1-\frac{\alpha}{\lambda}}{1-\lambda} \tilde{z}\tilde{g}_t. \tag{46}$$

In other words, inequality γ_t is *not* proportional to the output gap, *unless* tax incidence is *uniform*, i.e., $\alpha = \lambda$. This observation has key implications for our welfare analysis below.

By imposing $c_t = c_t^*$ in (46) we obtain the *flexible-price level of inequality*

$$\gamma_t^* = -\frac{1-\frac{\alpha}{\lambda}}{1-\lambda} \tilde{z}\tilde{g}_t \tag{47}$$

Notice that γ_t^* is not zero but a function of government spending. The sign of its response to g depends on tax incidence α/λ. An increase in spending accompanied by a transfer to the constrained H agents ($\alpha < \lambda$) reduces inequality, and viceversa. Crucially, and to the extent that insurance is imperfect, those movements in inequality generate an inefficiency even under flexible prices, i.e., even in the absence of a nominal rigidity distortion.

The policy tradeoff The above observations encapsulate the novel tradeoff that our paper identifies. When the tax incidence of government purchases is unequal ($\alpha \neq \lambda$), closing the output gap will also entail, for monetary policy, variations in inequality which are *inefficient,* insofar as they are associated with imperfect insurance (i.e., inequality in consumption).

On the other hand, perfect redistribution (or insurance) would imply $\gamma_t^{**} = 0$ and closing the gap with respect to an output level that is different from its flexible-price counterpart:

$$\underbrace{c_t^{**}}_{\substack{\text{perfect-insurance} \\ \text{consumption}}} = \underbrace{c_t^*}_{\substack{\text{flex-price} \\ \text{consumption}}} + \frac{1-\frac{\alpha}{\lambda}}{1-\chi} \tilde{z}\tilde{g}_t = -\frac{\chi - \frac{\alpha}{\lambda}}{\chi - 1} \tilde{z}\tilde{g}_t \tag{48}$$

The first-best would be to achieve perfect redistribution by implementing targeted, agent-specific transfers, but we assume those are unfeasible. In our second-best world, an attempt to stabilize aggregate activity around the level consistent with perfect insurance will trigger inflation movements and the inefficiency typically associated with nominal rigidities. Replacing c_t^{**} in the Phillips and IS curve respectively and assuming an exogenous AR(1) process for $\tilde{g}_t$ with persistence $p \in [0, 1]$, the value of inflation consistent with perfect insurance is :

$$\pi_t^{**} = \frac{\kappa}{1-\beta p} \frac{1-\frac{\alpha}{\lambda}}{1-\chi} \tilde{z}\tilde{g}_t. \tag{49}$$

Equation (49) shows that, unless $\alpha = \lambda$, it is never efficient (under perfect insurance) to implement a zero inflation equilibrium in response to fluctuations in government spending. That contrasts with the desired inflation rate under flexible prices, which is zero.

To better understand the link between government spending and inequality, and how that link generates the novel tradeoff we emphasize, it is useful to distinguish between the two polar cases of progressive vs. regressive taxation. Suppose first that the taxation used to finance public spending is *progressive* ($\alpha < \lambda$). This entails a transfer towards the high-MPC hand-to-mouth agents. This transfer, in turn, generates *three* distinct effects, as already highlighted in our analysis on pure transfers above. First, the transfer towards the H agents decreases *inequality* mechanically, through a direct effect. Second, for a given stance of monetary policy, the transfer towards the H agents is expansionary on aggregate demand, regardless of how individual incomes comove with the cycle, i.e., regardless of χ.[14] Third, the ensuing effect on inequality, however, depends on the value of χ. If the income of H over-reacts endogenously to the above increase in aggregate income ($\chi > 1$), inequality then falls further, amplifying the initial direct effect. To restore perfect insurance, the central bank needs to engineer a recession and deflation that increases the inequality back towards zero and thus closes the distributional gap. Conversely, if instead $\chi < 1$, the aggregate expansion leads endogenously to an increase in inequality back towards its optimal level (of zero); to close the gap entirely, the central bank needs to act procyclically, so as to further amplify the expansion (and inflate) up to the point where the direct effect through the transfer has been neutralized.

Evidently, all these results are overturned if spending is financed through *regressive* taxation, $\alpha > \lambda$: there is a transfer to savers, and inequality increases directly. There is an aggregate recession, which translates into a further increase in income inequality when $\chi > 1$ and a decrease when $\chi < 1$. In the former case, restoring perfect insurance thus requires engineering a reduction in inequality through an expansion and inflation. In the latter, the opposite.

There are thus *three relevant channels*: (i) the direct effect on income inequality through the progressivity or regressivity of taxes; (ii) the (indirect) effect of the former on aggregate demand, which depends on the equilibrium monetary policy response; (iii) the indirect effect on income inequality through its endogenous response to aggregate income.

To summarize, neutralizing fluctuations in inequality so as to attempt restoring perfect insurance requires using the central bank's leverage over real activity through inflation variations. Unless the incidence of taxation is uniform, there is a tension between stabilizing real activity so as to minimize *supply-side*, price-stickiness distortions (via a policy of zero inflation) and *demand-side*, imperfect-insurance distortions. As we will now see, optimal policy consists of balancing this tradeoff.

Welfare objective

In the Appendix, we derive a second-order approximation to the utilitarian welfare objective around the non-distorted steady state. We have the following proposition.

Proposition 6 *The welfare function is proportional to the quadratic loss $\mathcal{L}_t$:*

$$\mathcal{L}_t \equiv \frac{1}{2}\left\{\pi_t^2 + \underbrace{\frac{\sigma^{-1}+\tilde{\varphi}}{\psi}\left(c_t + \tilde{z}\tilde{g}_t\right)^2}_{output\ gap} + \underbrace{\frac{\sigma^{-1}\tilde{z}+\tilde{\sigma}_G^{-1}}{\psi}\tilde{g}_t^2}_{govt.\ spending} + \underbrace{\frac{(\sigma\tilde{z})^{-1}}{\psi}\lambda\left(1-\lambda\right)\gamma_t^2}_{inequality}\right\}. \tag{50}$$

The welfare objective now features four components. The first three components are common to a standard RANK model in which government spending provides utility, as, e.g., in Woodford (2011). The first two refer to losses from inflation and output gap movements (with the output gap being now affected by variations in government spending). The third component captures the loss from government spending deviating from the efficient benchmark, i.e., the "Samuelson condition" for efficient public good provision, $U_C^j(C) = U_G^j(G)$, that we assume holds in steady state.

The fourth, and novel component, is typical of our TANK framework, and captures quadratic losses from movements in *inequality*. What is novel here is that, unless taxation used to finance government spending is *uniform* ($\alpha = \lambda$), the inequality term is *not* proportional to the output gap relative to flexible prices—there is a *wedge* between the two, driven by fiscal redistribution, as apparent by direction inspection of (45) and discussed above. Put differently, the presence of transfers between agents ($\alpha \neq \lambda$) generates an endogenous wedge (proportional to inequality) between the welfare-relevant and the flexible-price level of output. Hence movements in inequality induced by government spending make the flexible-price equilibrium suboptimal.

[14] See the discussion in Bilbiie (2020) and Bilbiie, Monacelli, and Perotti (2012) for an earlier version of that result.

4.1 Optimal monetary-fiscal policy

In this section we lay out the optimal policy problem. We first study optimal policy assuming that government spending evolves *exogenously.* This assumption allows us to isolate the first channel we unveiled, i.e., fiscal redistribution as an additional constraint on monetary policy, affecting its stabilization capacity. Then we turn to studying the joint *optimal* design of fiscal and monetary policy when government spending is assumed to be the fiscal policy instrument. We focus again on policy under discretion.

Exogenous government spending

Notice that in the economy hitherto specified, if public spending could be optimally chosen it would be set to its (Samuelson-rule-optimal) steady-state level, i.e. $\tilde{g}_t^{opt} = 0$. This would allow the central bank to simultaneously close the output gap, attain zero inflation, and close the inequality gap. To elucidate the relevant tradeoffs, we begin by assuming that government spending evolves instead exogenously and characterize the optimal monetary policy response to such shocks.

The optimal policy problem in this case reduces to minimizing (50) subject to the static constraint (45) (which can be directly replaced in the objective function), and to (44), taking $\mathbb{E}_t \pi_{t+1}$ and $\tilde{g}_t$ as given for all t. The first order conditions lead to the optimal targeting rule:

$$\kappa \pi_t + \frac{\sigma^{-1} + \tilde{\varphi}}{\psi} \left[1 + \frac{\lambda (\chi - 1)^2}{(1-\lambda)\sigma \tilde{\varphi}} \right] (c_t + \tilde{z}\tilde{g}_t) + \left(1 - \frac{\alpha}{\lambda}\right) \frac{(\chi-1)\lambda}{1-\lambda} \frac{\sigma^{-1}}{\psi} \tilde{g}_t = 0 \tag{51}$$

Equation (51) together with the Phillips curve (44) fully determines the equilibrium.

Notice that in the benchmark case of equal incidence of taxation ($\alpha = \lambda$) or endogenous redistribution of income generating proportional incomes $\chi = 1$, it is optimal—and feasible—to close the output gap relative to flexible prices *and* simultaneously stabilize inflation in response to government spending shocks. In that particular case, the policy authority faces no inherent tradeoff in achieving the first-best allocation: in New Keynesian jargon, the efficient and the flexible-price level of activity coincide.

Away from the equal-incidence benchmark, however, government spending acts as a cost-push or pull term, depending on *redistribution* (i.e., on both its exogenous $1 - \frac{\alpha}{\lambda}$ and endogenous $\chi - 1$ dimensions). In other words, the *combination* of unequal incidence of taxation and cyclical inequality generates a time-varying wedge between the output gap measured relative to the efficient level of activity and the one measured relative to the flexible-price level of activity.

To understand the equilibrium dynamics triggered by the shock under optimal policy through its distributional implications, we solve the model (combining (51) and (44)) assuming that $\tilde{g}_t$ follows an AR1 process with persistence $p \in [0,1]$. This implies $\mathbb{E}_t \tilde{g}_{t+1} = p\tilde{g}_t$, which in turn allows to obtain the following expressions for the output gap and inflation respectively:

$$\begin{aligned} c_t + \tilde{z}\tilde{g}_t &= -\left(1 - \frac{\alpha}{\lambda}\right) \frac{\lambda}{1-\lambda} \frac{\chi - 1}{1 + \sigma\tilde{\varphi}} \tilde{\Phi} \tilde{g}_t; \\ \gamma_t &= -\left(1 - \frac{\alpha}{\lambda}\right)\left(1 + \frac{\kappa\theta}{1-\beta p}\right) \frac{\tilde{z}}{1-\lambda} \tilde{\Phi} \tilde{g}_t \\ \pi_t &= -\left(1 - \frac{\alpha}{\lambda}\right) \frac{\lambda}{1-\lambda} \frac{\chi - 1}{1 + \sigma\tilde{\varphi}} \frac{\kappa}{1-\beta p} \tilde{\Phi} \tilde{g}_t, \end{aligned} \tag{52}$$

where we defined $\tilde{\Phi} \equiv \left[1 + \frac{\kappa\theta}{1-\beta p} + \frac{1}{\sigma\tilde{\varphi}} \frac{\lambda(\chi-1)^2}{1-\lambda}\right]^{-1} > 0$.

The intuition again parallels that introduced in Section (3). The effect of an increase in government spending on inflation depends on the sign of the composite parameter summarizing the effect of exogenous (through tax incidence) and endogenous redistribution $\left(1 - \frac{\alpha}{\lambda}\right)(\chi - 1)$. An increase in spending financed with *progressive* taxes ($\alpha < \lambda$), i.e., with higher incidence on the upper part of the income and wealth distribution, entails a transfer to H agents that reduces inequality through a direct effect. When income inequality is endogenously countercyclical ($\chi > 1$), the ensuing aggregate expansion is associated to a further reduction in inequality. Optimal policy requires counteracting this and engineering a deflation: a cost-pull shock. Notice that this implies a negative output gap relative to the flexible-price level $c_t^* = -\tilde{z}\tilde{g}_t$, yet a *positive* output gap relative to the *perfect-insurance level* c_t^{**} given in (48):

$$c_t - c_t^{**} = \frac{1 - \frac{\alpha}{\lambda}}{\chi - 1} \left(1 + \frac{\kappa\theta}{1-\beta p}\right) \tilde{\Phi} \tilde{z} \tilde{g}_t.$$

Naturally, with spending financed by regressive taxation $\alpha > \lambda$ (and still endogenously countercyclical income

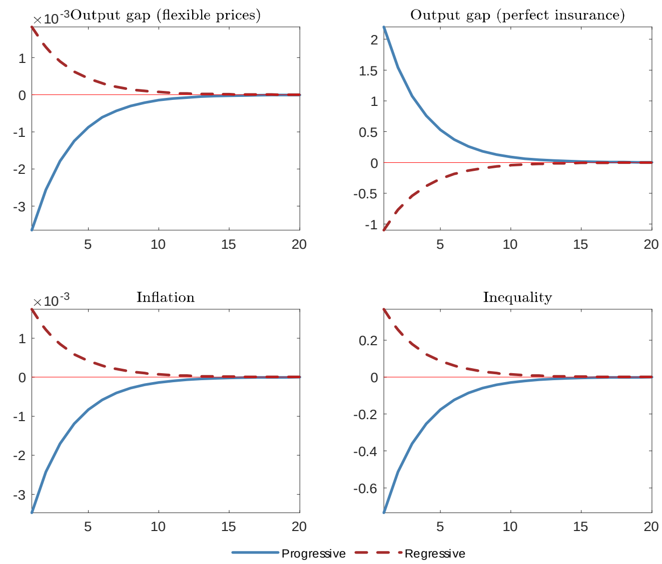

Figure 1: Impulse responses to a government spending shock under the optimal policy: progressive ($\alpha < \lambda$) vs. regressive ($\alpha > \lambda$) taxation. Note: throughout we assume counter-cyclical inequality, $\chi > 1$.

inequality $\chi > 1$) the mirroring reasoning holds and spending shocks are inflationary, i.e., *cost-push*. While when income inequality is instead procyclical, the dynamics triggered be progressive versus regressive taxation are simply inverted: in other words, the sufficient statistic is indeed the convolution of exogenous and endogenous redistribution expressed by the term $\left(1 - \frac{\alpha}{\lambda}\right)(\chi - 1)$.

Figure 1 illustrates the above results by displaying the effects on the (welfare-relevant) output gap, inflation and inequality of an exogenous rise of government spending g_t under the optimal policy, for a baseline *parameterization* with quarterly discount factor $\beta = .99$, elasticity of substitution between goods $\theta = 6$, share of H agents $\lambda = .4$ paying share of taxes $\alpha = .3$, inverse labor elasticity $\varphi = 3$, elasticity of H's net income to aggregate income $\chi = 1.2$ and price adjustment cost parameter $\psi = 69.9$ (yielding a Calvo-equivalent duration of one year).

In our baseline case we set $\alpha < \lambda$ (i.e., progressive taxation), but we compare it with the case of regressive taxation below. We also assume that *income inequality is countercyclical*, i.e., $\chi > 1$; in the converse case, all the predictions are overturned since government spending then has a negative consumption multiplier. In light of the evidence put forward in Patterson (2019), however, the case of countercyclical income inequality assumed as a baseline is the most empirically plausible.

The baseline case of progressive taxation ($\alpha < \lambda$, solid line) is compared to the one of regressive taxation ($\alpha > \lambda$, dashed line). In the case of progressive taxation, a positive government spending shock, because of the implicit transfer, creates an inefficient redistribution of income from unconstrained to constrained agents and a too large aggregate demand expansion. Optimal policy contains this expansion, thereby generating some deflation. The shock thus looks like a "cost-pull" shock in the standard New Keynesian taxonomy. At the same time, under progressive taxation, the increase in government spending accompanied by a transfer to the constrained H agents induces a fall in inequality.

Recall, from (46), that inequality is proportional to both the output gap and government spending. The sign of this proportionality depends on χ (the elasticity of H's income to aggregate income) and on the tax incidence factor α/λ. Under the assumption $\chi > 1$, the rise in economic activity reduces inequality. This effect is strengthened by the expansion in government spending in the case of progressive taxation ($\alpha < \lambda$).

The mirror reasoning holds for government spending shocks that are regressive, i.e., accompanied by a transfer from constrained to unconstrained agents ($\alpha > \lambda$), which instead act as cost-push disturbances. In this case optimal policy calls for an increase in inflation and a corresponding contraction in real activity relative to the welfare-relevant output target (although an expansion relative to the natural, flexible-price level of output). In all cases, optimal policy calls for movements in inequality away from the efficient steady state benchmark of zero inequality and full insurance.

Endogenous fiscal policy

So far we have treated government spending as evolving exogenously. We turn next to a setup in which government spending is itself chosen optimally, in conjunction with monetary policy. This case illustrates that the optimal policy mix is a combination of aggregate demand stabilization and redistribution via monetary and fiscal policy, with implications for inequality.

We assume, without loss of generality, that the economy is hit by an exogenous additive cost-push disturbance u_t so that the Phillips-curve equation becomes:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left(c_t + \widetilde{z} \widetilde{g}_t \right) + u_t \tag{53}$$

In this setting monetary and fiscal policy are chosen simultaneously in order to stabilize the exogenous (tradeoff-inducing) shock u_t which evolves independent of policy. The resulting optimal policy problem is to minimize (50), having replaced (45), subject to (53) taking $\mathbb{E}_t \pi_{t+1}$ as given for all t.

There are two optimality conditions: the same first-order condition obtained previously (51) and a new one for the optimal choice of $\tilde{g}_t$:

$$\kappa \widetilde{z} \pi_t + \widetilde{z} \frac{\sigma^{-1} + \tilde{\varphi}}{\psi} \left(c_t + \widetilde{z} \widetilde{g}_t \right) + \frac{\sigma^{-1} \widetilde{z} + \tilde{\sigma}_G^{-1}}{\psi} \tilde{g}_t + \frac{\chi - \frac{\alpha}{\lambda}}{\chi - 1} \frac{\sigma^{-1}}{\psi} \frac{\lambda \left(\chi - 1 \right)^2}{1 - \lambda} \left(c_t + \frac{\chi - \frac{\alpha}{\lambda}}{\chi - 1} \widetilde{z} \widetilde{g}_t \right) = 0. \tag{54}$$

Using (51), the above condition becomes:

$$\tilde{g}_t = - \left(1 - \frac{\alpha}{\lambda} \right) \left(\chi - 1 \right) \frac{\lambda}{1 - \lambda} \frac{\tilde{\sigma}_G}{\widetilde{z} \tilde{\sigma}_G + \sigma} \left(c_t + \frac{\chi - \frac{\alpha}{\lambda}}{\chi - 1} \widetilde{z} \widetilde{g}_t \right),$$

which implies that optimal government spending (in log deviation) is proportional to the inequality gap. The sign of the comovement is driven once again by the sufficient statistic capturing the effect of redistribution discussed above $\left(1 - \frac{\alpha}{\lambda}\right)(\chi - 1)$ (i.e., both the progressivity of taxes α/λ and the endogenous redistribution of income χ). This is consistent with our result on optimal transfers, in particular we have $\tilde{g}_t = 0$ when taxes have uniform incidence.

Using the AR(1) structure as above we obtain the optimal value of government spending:

$$\tilde{g}_t^{opt} = \left(1 - \frac{\alpha}{\lambda} \right) \left(\chi - 1 \right) \Psi u_t \tag{55}$$

where $\Psi \equiv \frac{\lambda}{1-\lambda} \tilde{\Phi} \frac{\theta}{1-\beta p} \left[\widetilde{z} + \sigma \tilde{\sigma}_G^{-1} + \left(1 - \frac{\alpha}{\lambda}\right)^2 \frac{\lambda}{1-\lambda} \left(1 + \frac{\kappa\theta}{1-\beta p}\right) \tilde{\Phi} \widetilde{z} \right]^{-1} > 0$.

Replacing (55) in the first-order condition (54) we also obtain closed-form solutions for the output gap relative to the flexible-price output, inequality, as well as the output gap relative to the perfect-insurance output:

$$\begin{aligned}
\underbrace{c_t - c_t^*}_{\substack{\text{output gap} \\ \text{relative to flex-price} \\ \text{output}}} &= c_t + \widetilde{z} \widetilde{g}_t = - \left[\frac{1 - \lambda}{\lambda} \left(\widetilde{z} + \sigma \tilde{\sigma}_G^{-1} \right) + \widetilde{z} \left(1 - \frac{\alpha}{\lambda} \right)^2 \right] \Psi u_t \\
\underbrace{c_t - c_t^{**}}_{\substack{\text{output gap} \\ \text{relative to perfect–insurance} \\ \text{output}}} &= - \frac{1 - \lambda}{\lambda} \left(\widetilde{z} + \sigma \tilde{\sigma}_G^{-1} \right) \Psi u_t \\
\underbrace{\gamma_t}_{\text{inequality}} &= \frac{1 - \chi}{1 - \lambda} \left(c_t - c_t^{**} \right) = \frac{\chi - 1}{\lambda} \left(\widetilde{z} + \sigma \tilde{\sigma}_G^{-1} \right) \Psi u_t
\end{aligned}$$

In the equal-incidence benchmark corresponding to either $\alpha = \lambda$ or $\chi = 1$ optimal public spending is constant ($\tilde{g}_t^{opt} = 0$). Otherwise equation (55) shows that government spending is *used actively* (away from the steady-state Samuelson condition of efficient provision of public goods), together with monetary policy, to stabilize cost-push shocks.

The *sign* of the optimal response of public spending, however, depends again on the sign of the sufficient-statistic redistribution wedge $\left(1 - \frac{\alpha}{\lambda}\right)(\chi - 1)$. Figure 2 illustrates the alternative cases. The figure plots impulse responses to a positive cost-push shock under the optimal policy. We assume the cost-push term u_t evolves as an AR1 process with persistence equal to 0.9. Otherwise, we employ the same calibration as for Figure 1.

Consider first the baseline case of $\alpha < \lambda$ with $\chi > 1$ (progressive taxation and countercyclical H's income share). In light of a shock that triggers a recession and inflation, optimal policy calls for a *counter-cyclical* expansion in government spending. The spending instrument, when accompanied by a transfer to constrained agents which boosts aggregate demand disproportionately more, is a more effective way to mitigate the recession.[15] The detailed intuition is as follows, and draws again on the core intuition developed in Section (3) and in the previous section. In a nutshell, government

[15] A similar counter-cyclical expansion of government spending is also obtained in the case of regressive taxation and pro-cyclical H's income, which however we deem as an empirically less plausible case.

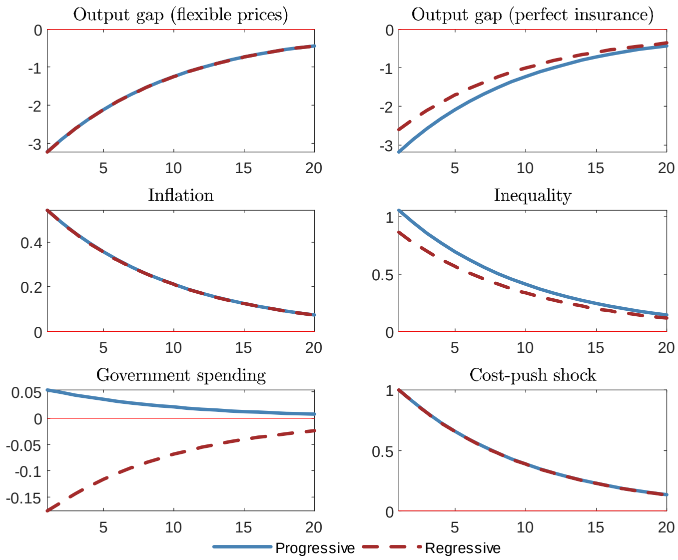

Figure 2: Impulse responses to a cost-push shock under the optimal policy. Note: solid line progressive taxation ($\alpha < \lambda$) , dashed line regressive taxation ($\alpha > \lambda$).

spending is "cost-pull" and is thus optimally used to counteract a cost-push shock. Without the spending increase, the cost-push shock generates a recession and, with $\chi > 1$, an increase in inequality that opens a novel inefficiency gap. With progressive taxation, the increase in government spending features the implicit transfer component that helps mitigate that gap directly. Furthermore, the positive multiplier effect of spending *itself* generates an aggregate expansion that further (and indirectly) mitigates the inefficiency gap.

However, the public spending instrument can also be optimally used in a *pro-cyclical* way. In the case in which public spending is financed via regressive taxes $\alpha > \lambda$ and H's income share is counter-cyclical ($\chi > 1$), a *reduction* in public spending is part of the optimal policy mix. The reason is that, in the case of regressive taxes, lower public spending boosts aggregate consumption, due to an implicit transfer. Notice that a similar reasoning applies to the case of progressive taxation coupled with pro-cyclical H's income (not shown). In that case, as well, the consumption multiplier of spending is negative, therefore optimal policy calls for a contraction in government spending in light of an ensuing recession.

5 Conclusions

In a heterogenous agent economy, imperfect insurance and nominal rigidities interact to generate a *novel channel* through which monetary and fiscal policy are linked in their optimal design. With imperfect insurance, movements in inequality are inherently inefficient. A lump-sum transfer to financially constrained agents reduces inequality, and viceversa. However those variations in inequality generate an inefficiency even under flexible prices, i.e., even in the absence of a nominal rigidity distortion. In order to restore perfect insurance the policymaker must leverage over real activity through inflation variations, which however are costly due nominal rigidities. There is therefore a inherent tension between stabilizing real activity so as to minimize supply-side, price-stickiness distortions and demand-side, imperfect-insurance distortions.

The financing of government spending naturally generates room for transfers across agents. Public spending financed with either progressive or regressive taxation therefore opens up an inefficient *inequality gap*. To close it, the planner must use its leverage over inequality through affecting aggregate demand, which entails exploiting nominal rigidities—and thus generating inefficient inflation variations. A novel tradeoff thus occurs: replicating the flexible-price equilibrium now entails deviations from the first-best benchmark of perfect-insurance; while eliminating movements in inequality altogether—in an attempt to achieve perfect-insurance—entails instead movements in inflation and thus the inefficiencies customarily associated with nominal rigidities.

Optimal policy tries to resolve this tradeoff. With exogenous public spending, optimal policy implies that public spending with unequal incidence of taxation has either a cost-push or cost-pull dimension, depending on whether taxation is progressive or regressive, and on whether income inequality is pro- or countercyclical; indeed, we define a tax-incidence sufficient statistic that is a convolution of these two features and governs the optimal-policy properties. To give one (most empirically-plausible) example, a spending increase financed with progressive taxes entails a transfer to the hand-to-mouth agents that reduces inequality directly. If income inequality is countercyclical, the ensuing expansion further reduces inequality. To contain this inefficient inequality gap, the central bank reacts optimally by engineering a recession and deflation, i.e., a cost-pull shock.

When public spending is itself endogenous and chosen optimally, we show that it is always used, insofar as its tax incidence is not uniform, to neutralize cost-push shocks that generate stabilization tradeoffs. In the empirically plausible case of progressive taxation and countercyclical inequality, government spending is optimally increased in an attempt to contain cost-push shocks. That is, the cost-pull dimension of spending is then used in a counter-cyclical fashion.

Our general point is that hitherto seemingly orthogonal features of the economy should become key inputs in monetary policy design, and call for its coordination with fiscal policy for stabilization and redistribution. Namely, the progressivity of taxation used to finance public spending and the incidence of aggregate fluctuations on individual incomes (the cyclicality of inequality) are paramount inputs in designing the optimal mix of monetary and fiscal policies.

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Online Appendix to Bilbiie, Monacelli, and Perotti:

"Stabilization vs. Redistribution: The Optimal Monetary-Fiscal Mix"

A Derivations

We approximate the economy around an *efficient* steady-state, whereby the optimal subsidy inducing zero profits in steady state *also* implies that consumption shares are equalized across agents—thereby inducing full insurance. In particular, the fiscal authority subsidizes sales at the constant rate τ^S and redistributes the proceedings in a lump-sum fashion T^S such that in steady-state there is marginal cost pricing, and profits are zero. The profit function becomes $D_t(k) = \left(1+\tau^S\right) P_t(k) Y_t(k) - W_t N_t(k) - \frac{\psi}{2}\left(\frac{P_t(k)}{P_{t-1}^{**}} - 1\right)^2 P_t Y_t + T_t^F$ where by balanced budget $T_t^F = \tau^S P_t(k) Y_t(k)$. Efficiency requires $\tau^S = (\theta-1)^{-1}$, such that under flexible prices $P_t^*(k) = W_t^*$ and hence profits are $D_t^* = 0$ (with sticky prices profits are not zero as the mark-up is not constant). Under this assumption we have that in steady-state:

$$\frac{U_N\left(N^H\right)}{U_C\left(C^H\right)} = \frac{U_N\left(N^S\right)}{U_C\left(C^S\right)} = \frac{W}{P} = 1 = \frac{Y}{N},$$

where $N^j = N = Y$ and $C^j = C = Y - G$.

$$U_t = U(C_t, N_t, G_t) = \frac{C_t^{1-\sigma^{-1}} - 1}{1-\sigma^{-1}} - \chi \frac{N^{1+\varphi}}{1+\varphi} + \chi_G \frac{G_t^{1-\sigma_G^{-1}} - 1}{1-\sigma_G^{-1}}, \tag{56}$$

$$\begin{aligned} U_C w + U_N &= 0 \\ \frac{\chi N^{\varphi}}{C^{-\sigma^{-1}}} &= \frac{W}{P} = 1 = \frac{Y}{N} = \frac{C+G}{N}, \\ U_C &= -U_N \end{aligned}$$

$$U_G = U_C \rightarrow C^{-\sigma^{-1}} = \chi_G G^{-\sigma_G^{-1}}$$

It is useful to remember that the Taylor series expansion of a function $f(x_t)$ around a given value $f(x)$ is:

$$\frac{f(x_t) - f(x)}{f(x)} = \frac{f_x x}{f} \hat{x}_t + \frac{1}{2} \frac{f_{xx} x^2}{f} \hat{x}_t^2$$

For instance:

$$\frac{X_t - X}{X} = x_t + \frac{1}{2} x_t^2 \text{ where } x_t = \ln \frac{X_t}{X}$$

Suppose further that the social planner maximizes a convex combination of the utilities of the two types, weighted by the mass of agents of each type: $U_t(.) \equiv \lambda U^H\left(C_t^H, N_t^H\right) + [1-\lambda] U^S\left(C_t^S, N_t^S\right)$. The second-order approximation to type $j's$ utility around the steady state delivers:

$$\begin{aligned} \hat{U}_{j,t} &\equiv U_j\left(C_{j,t}, N_{j,t}\right) - U_j\left(C_j, N_j\right) = \\ &= U_C C^j \left[c_t^j + \frac{1-\sigma^{-1}}{2}\left(c_t^j\right)^2\right] - U_N N^j \left[n_t^j + \frac{1+\varphi}{2}\left(n_t^j\right)^2\right] \\ &+ U_G G\left[g_t + \frac{1}{2}\frac{U_{GG} G}{U_G} g_t^2\right] + t.i.p + O\left(\parallel \zeta \parallel^3\right), \end{aligned} \tag{57}$$

where for any variable $x_t \equiv \ln(X_t/X)$ and $\hat{g}_t \equiv \ln(G_t/G)$. Writing instead with g expressed in steady-state Y units $g_t \equiv \frac{G}{Y}\hat{g}_t$

$$\begin{aligned} \hat{U}_{j,t} &\equiv U_j\left(C_{j,t}, N_{j,t}\right) - U_j\left(C_j, N_j\right) = \\ &= U_C C^j \left[c_t^j + \frac{1-\sigma^{-1}}{2}\left(c_t^j\right)^2\right] - U_N N^j \left[n_t^j + \frac{1+\varphi}{2}\left(n_t^j\right)^2\right] \end{aligned} \tag{58}$$

$$+ U_G Y\left[g_t + \frac{1-\sigma_G^{-1}}{2}\frac{Y}{G} g_t^2\right] + t.i.p + O\left(\parallel \zeta \parallel^3\right), \tag{59}$$

Approximating the goods market clearing condition to second order delivers:

$$
\begin{aligned}
&\lambda C_{H,t} + (1-\lambda) C_{S,t} + G_t \\
&\simeq \lambda C c_{H,t} + (1-\lambda) C c_{S,t} + Y g_t + \frac{1}{2}\left(\lambda C c_{H,t}^2 + (1-\lambda) C c_{S,t}^2 + Y g_t^2\right) \\
&= \left(\lambda N_{H,t} + (1-\lambda) N_{S,t}\right)\left(1 - \frac{\psi}{2}\pi_t^2\right) \\
&\simeq \lambda N n_{H,t} + (1-\lambda) N n_{S,t} + \frac{1}{2}\left(\lambda N n_{H,t}^2 + (1-\lambda) N n_{S,t}^2\right) - \frac{\psi\pi}{1-\frac{\psi}{2}\pi^2}\pi_t - \frac{1}{2}\frac{\psi}{1-\frac{\psi}{2}\pi^2}\pi_t^2
\end{aligned}
$$

It is straightforward to show that the optimal long-run inflation target in this economy is, just like in a typical RANK model, $\pi = 0$. Replacing, the linearly-aggregated first-order term is thus found from this second-order approximation of the economy resource constraint as:

$$
\begin{aligned}
&\lambda \frac{C}{Y} c_{H,t} + (1-\lambda)\frac{C}{Y} c_{S,t} + g_t + \frac{1}{2}\left(\lambda \frac{C}{Y} c_{H,t}^2 + (1-\lambda)\frac{C}{Y} c_{S,t}^2 + g_t^2\right) \\
&= \lambda n_{H,t} + (1-\lambda) n_{S,t} + \frac{1}{2}\left(\lambda n_{H,t}^2 + (1-\lambda) n_{S,t}^2\right) - \frac{\psi}{2}\pi_t^2
\end{aligned}
$$

where the last term captures the welfare cost of inflation.

$$
\begin{aligned}
&\lambda \frac{C}{Y} c_{H,t} + (1-\lambda)\frac{C}{Y} c_{S,t} + g_t - \lambda n_{H,t} - (1-\lambda) n_{S,t} \\
&+ \frac{1}{2}\left(\lambda \frac{C}{Y} c_{H,t}^2 + (1-\lambda)\frac{C}{Y} c_{S,t}^2 + g_t^2 - \left(\lambda n_{H,t}^2 + (1-\lambda) n_{S,t}^2\right)\right) \\
&= -\frac{\psi}{2}\pi_t^2
\end{aligned} \tag{60}
$$

Note that since $U_C C^j$ and $U_N N^j$ are equal across agents we can aggregate the approximations of individual utilities above (57), using (60) to eliminate linear terms, into:

$$
\hat{U}_t = U_C Y\left[\lambda \frac{C}{Y} c_t^H + (1-\lambda)\frac{C}{Y} c_t^S + \frac{1-\sigma^{-1}}{2}\frac{C}{Y}\left[\lambda\left(c_t^H\right)^2 + (1-\lambda)\left(c_t^S\right)^2\right]\right] \tag{61}
$$

$$
- U_C N\left[\lambda n_t^H + (1-\lambda) n_t^S + \frac{1+\varphi}{2}\left[\lambda\left(n_t^H\right)^2 + (1-\lambda)\left(n_t^S\right)^2\right]\right] \tag{62}
$$

$$
+ U_G Y\left[g_t + \frac{1-\sigma_G^{-1}}{2}\frac{Y}{G} g_t^2\right] + t.i.p + O\left(\| \zeta \|^3\right), \tag{63}
$$

Next, use a second order approximation of the aggregate resource constraint to eliminate linear term and corresponding quadratic terms

$$
\begin{aligned}
\hat{U}_t = &-U_C Y\left\{\frac{\sigma^{-1}}{2}\frac{C}{Y}\left[\lambda\left(c_t^H\right)^2 + (1-\lambda)\left(c_t^S\right)^2\right] + \frac{\varphi}{2}\left[\lambda\left(n_t^H\right)^2 + (1-\lambda)\left(n_t^S\right)^2\right] + \frac{\sigma_G^{-1}}{2}\frac{Y}{G} g_t^2 + \frac{\psi}{2}\pi_t^2\right\} \\
&+ t.i.p + O\left(\| \zeta \|^3\right).
\end{aligned}
$$

Letting $\tilde{g}_t \equiv \frac{1}{1-G_Y} g_t; \tilde{\sigma}_G^{-1} \equiv \frac{1-G_Y}{G_Y}\sigma_G^{-1}; \tilde{\varphi} \equiv (1-G_Y)\varphi$.

$$
\begin{aligned}
\hat{U}_t = &-U_C C\left\{\frac{\sigma^{-1}}{2}\left[\lambda\left(c_t^H\right)^2 + (1-\lambda)\left(c_t^S\right)^2\right] + \frac{\varphi}{2(1-G_Y)}\left[\lambda\left(n_t^H\right)^2 + (1-\lambda)\left(n_t^S\right)^2\right] + \frac{\tilde{\sigma}_G^{-1}}{2}\tilde{g}_t^2 + \frac{\psi}{2}\pi_t^2\right\} \\
&+ t.i.p + O\left(\| \zeta \|^3\right).
\end{aligned}
$$

Quadratic terms can be expressed as a function of aggregate consumption (output). Notice that in evaluating these quadratic terms we can use first-order approximations of the optimality conditions (higher order terms imply terms of order $O\left(\| \zeta \|^3\right)$).

$$c_t^H = \chi c_t + z\left(\chi - \frac{\alpha}{\lambda}\right)\tilde{g}_t$$

$$c_t^S = \frac{1-\lambda\chi}{1-\lambda}c_t - \frac{\lambda}{1-\lambda}z\left(\chi - \frac{\alpha}{\lambda}\right)\tilde{g}_t$$

$$w_t = \left(\sigma^{-1} + \tilde{\varphi}\right)c_t + \tilde{\varphi}\tilde{g}_t$$

Using $(1-G_Y)\varphi = \tilde{\varphi}$

$$\begin{aligned}
n_t^H &= \varphi^{-1}w_t - \varphi^{-1}\sigma^{-1}c_t^H \\
&= \varphi^{-1}\left(\left(\sigma^{-1}+\tilde{\varphi}\right)c_t + \tilde{\varphi}\tilde{g}_t\right) - \varphi^{-1}\sigma^{-1}c_t^H \\
&= (1-G_Y)\left(\left[1+(\tilde{\varphi}\sigma)^{-1}(1-\chi)\right]c_t + \left[1-(\tilde{\varphi}\sigma)^{-1}z\left(\chi-\frac{\alpha}{\lambda}\right)\right]\tilde{g}_t\right)
\end{aligned}$$

$$\begin{aligned}
n_t^S &= \frac{1}{1-\lambda}n_t - \frac{\lambda}{1-\lambda}n_t^H \\
&= \frac{1-G_Y}{1-\lambda}\left(c_t+\tilde{g}_t\right) - \frac{\lambda}{1-\lambda}n_t^H \\
&= (1-G_Y)\left[1-\frac{\lambda}{1-\lambda}(\tilde{\varphi}\sigma)^{-1}(1-\chi)\right]c_t + (1-G_Y)\left[1+\frac{\lambda}{1-\lambda}(\tilde{\varphi}\sigma)^{-1}z\left(\chi-\frac{\alpha}{\lambda}\right)\right]\tilde{g}_t
\end{aligned}$$

Replace in per-period loss, consider the "real" part (abstract from inflation) only, for notational convenience:

$$\begin{aligned}
&\frac{\sigma^{-1}}{2}\left[\lambda\left(\chi c_t + z\left(\chi-\frac{\alpha}{\lambda}\right)\tilde{g}_t\right)^2 + (1-\lambda)\left(\frac{1-\lambda\chi}{1-\lambda}c_t - \frac{\lambda}{1-\lambda}z\left(\chi-\frac{\alpha}{\lambda}\right)\tilde{g}_t\right)^2\right] \\
&+\frac{\tilde{\varphi}}{2}\left[\begin{array}{c}
\lambda\left(\left[1+(\tilde{\varphi}\sigma)^{-1}(1-\chi)\right]c_t + \left[1-(\tilde{\varphi}\sigma)^{-1}z\left(\chi-\frac{\alpha}{\lambda}\right)\right]\tilde{g}_t\right)^2 \\
+(1-\lambda)\left(\left[1-\frac{\lambda}{1-\lambda}(\tilde{\varphi}\sigma)^{-1}(1-\chi)\right]c_t + \left[1+\frac{\lambda}{1-\lambda}(\tilde{\varphi}\sigma)^{-1}z\left(\chi-\frac{\alpha}{\lambda}\right)\right]\tilde{g}_t\right)^2
\end{array}\right] + \frac{\tilde{\sigma}_G^{-1}}{2}\tilde{g}_t^2
\end{aligned}$$

Rearranging:

$$\begin{aligned}
&\frac{\sigma^{-1}}{2}\left[\left(1+\frac{\lambda}{1-\lambda}(\chi-1)^2\right)c_t^2 + z^2\left(\chi-\frac{\alpha}{\lambda}\right)^2\frac{\lambda}{1-\lambda}\tilde{g}_t^2 + 2\frac{\lambda}{1-\lambda}z\left(\chi-\frac{\alpha}{\lambda}\right)(\chi-1)c_t\tilde{g}_t\right] \\
&+\frac{\tilde{\varphi}}{2}\left[\begin{array}{c}
\left(1+\frac{\lambda}{1-\lambda}\left[(\tilde{\varphi}\sigma)^{-1}(\chi-1)\right]^2\right)c_t^2 \\
+\left(1+\frac{\lambda}{1-\lambda}\left[(\tilde{\varphi}\sigma)^{-1}z\left(\chi-\frac{\alpha}{\lambda}\right)\right]^2\right)\tilde{g}_t^2 \\
+2\left[1+\frac{\lambda}{1-\lambda}z(\tilde{\varphi}\sigma)^{-2}(\chi-1)\left(\chi-\frac{\alpha}{\lambda}\right)\right]c_t\tilde{g}_t
\end{array}\right] \\
&+\frac{\tilde{\sigma}_G^{-1}}{2}\tilde{g}_t^2
\end{aligned}$$

Multiply by 2 for simplicity:

$$\begin{aligned}
&\left(\sigma^{-1}+\tilde{\varphi}+\sigma^{-1}\frac{\lambda}{1-\lambda}(\chi-1)^2\left(1+(\tilde{\varphi}\sigma)^{-1}\right)\right)c_t^2 \\
&+\left(\tilde{\varphi}+\sigma^{-1}\frac{\lambda}{1-\lambda}z^2\left(\chi-\frac{\alpha}{\lambda}\right)^2\left(1+(\tilde{\varphi}\sigma)^{-1}\right)\right)\tilde{g}_t^2 \\
&+2\left[\tilde{\varphi}+\sigma^{-1}\frac{\lambda}{1-\lambda}z\left(1+(\tilde{\varphi}\sigma)^{-1}\right)(\chi-1)\left(\chi-\frac{\alpha}{\lambda}\right)\right]c_t\tilde{g}_t \\
&+\tilde{\sigma}_G^{-1}\tilde{g}_t^2
\end{aligned}$$

Use $z = \left[1 + (\tilde{\varphi}\sigma)^{-1}\right]^{-1}$:

$$\begin{aligned}
&\left(\sigma^{-1} + \tilde{\varphi} + \sigma^{-1}\frac{\lambda}{1-\lambda}(\chi - 1)^2\left(1 + (\tilde{\varphi}\sigma)^{-1}\right)\right)c_t^2 \\
&+ \left(\tilde{\varphi} + \sigma^{-1}\frac{\lambda}{1-\lambda}\left(\chi - \frac{\alpha}{\lambda}\right)^2\frac{1}{1 + (\tilde{\varphi}\sigma)^{-1}}\right)\tilde{g}_t^2 \\
&+ 2\left[\tilde{\varphi} + \sigma^{-1}\frac{\lambda}{1-\lambda}(\chi - 1)\left(\chi - \frac{\alpha}{\lambda}\right)\right]c_t\tilde{g}_t \\
&+ \tilde{\sigma}_G^{-1}\tilde{g}_t^2
\end{aligned}$$

$$\begin{aligned}
&\left(\sigma^{-1} + \tilde{\varphi}\right)\left(c_t^2 + \frac{\tilde{\varphi}}{\tilde{\varphi} + \sigma^{-1}}\tilde{g}_t^2 + 2\frac{\tilde{\varphi}}{\tilde{\varphi} + \sigma^{-1}}c_t\tilde{g}_t + \left(\frac{\tilde{\varphi}}{\tilde{\varphi} + \sigma^{-1}}\right)^2\tilde{g}_t^2 - \left(\frac{\tilde{\varphi}}{\tilde{\varphi} + \sigma^{-1}}\right)^2\tilde{g}_t^2\right) \\
&+ \sigma^{-1}\frac{\lambda}{1-\lambda}\frac{\tilde{\varphi} + \sigma^{-1}}{\tilde{\varphi}}\left((\chi - 1)^2 c_t^2 + 2(\chi - 1)\left(\chi - \frac{\alpha}{\lambda}\right)\frac{\tilde{\varphi}}{\tilde{\varphi} + \sigma^{-1}}c_t\tilde{g}_t + \left(\chi - \frac{\alpha}{\lambda}\right)^2\left(\frac{\tilde{\varphi}}{\tilde{\varphi} + \sigma^{-1}}\right)^2\tilde{g}_t^2\right) \\
&+ \tilde{\sigma}_G^{-1}\tilde{g}_t^2
\end{aligned}$$

The "real" part of the loss function becomes:

$$\begin{aligned}
&\left(\sigma^{-1} + \tilde{\varphi}\right)\left(c_t + \frac{\tilde{\varphi}}{\tilde{\varphi} + \sigma^{-1}}\tilde{g}_t\right)^2 + \left(\sigma^{-1}\frac{\tilde{\varphi}}{\tilde{\varphi} + \sigma^{-1}} + \tilde{\sigma}_G^{-1}\right)\tilde{g}_t^2 \\
&+ \sigma^{-1}\frac{\lambda}{1-\lambda}\frac{\tilde{\varphi} + \sigma^{-1}}{\tilde{\varphi}}\left((\chi - 1)c_t + \left(\chi - \frac{\alpha}{\lambda}\right)\frac{\tilde{\varphi}}{\tilde{\varphi} + \sigma^{-1}}\tilde{g}_t\right)^2
\end{aligned}$$

where recall inequality is the last term in brackets

$$\gamma_t = -\frac{1}{1-\lambda}\left((\chi - 1)c_t + z\left(\chi - \frac{\alpha}{\lambda}\right)\tilde{g}_t\right)$$

Rewriting, this yields (50) in the text.

B Derivation of loss function with a distorted steady state, small distortion

In this Appendix, we extend the basic second-order approximation to welfare to the case of a "distorted" steady state, where "distorted" refers to the cross-sectional, inequality dimension. We consider the case of a "small" distortion, in the sense that fiscal policy does much of the redistribution in the long run but there is a first-order term left that might affect the monetary authority's job.

To isolate this point, we consider a simpler economy: we abstract from government spending and fiscal transfers used for stabilization purposes (if a transfer were available, it would of course be used to perfectly close this new first-order relevant welfare gap). Furthermore, for simplicity we consider the case of a unionized labor market following the references described in text for TANK models (i.e. Ascari et al, 2017; Bilbiie and Ragot, 2020; Galí et al, 2007). The equilibrium implication is that hours worked by each agent are identical and determined by an aggregate hours schedule:

$$N_t^H = N_t^S = N_t,$$

$$w_t = \xi (N_t)^{\varphi} (C_t)^{\sigma^{-1}}.$$

We maintain the by now well-understood channel of profit redistribution over the cycle (and its influence on the cyclicality of inequality emphasized in the benchmark case), and still assume that part of the profits may be redistributed to the hand-to-mouth. The budget constraints for the two households are, having imposed that no assets are held in equilibrium by the savers:

$$\begin{aligned}C_t^H &= W_t N_t + \frac{\tau^D}{\lambda} D_t - T^H \\ C_t^S &= W_t N_t + \frac{1-\tau^D}{1-\lambda} D_t - T^S,\end{aligned}$$

where T^j are constant transfers used to control steady-state inequality.

Steady state. Differently from the benchmark case of an efficient steady state (including perfect insurance), we consider an economy with arbitrary steady-state inequality. To model this, we now assume that the monopoly distortion is still eliminated by a sales subsidy (this allows us to abstract from the well-understood New Keynesian distortion); but this subsidy is now financed by taxing *both* households, in an arbitrary way. Specifically, let η be the fraction of the total subsidy for sales that is financed by taxes on the H group, $T^H = \frac{\eta}{\lambda}\tau^S Y$, with $T^S = \frac{1-\eta}{1-\lambda}\tau^S Y$.

Recall that profits in steady state (with zero inflation) are

$$D = \left(1+\tau^S - W\right) Y = \left(1+\tau^S\right)\frac{1}{\theta} Y$$

using the pricing equation which implies that the real wage is $W = \left(1+\tau^S\right)\frac{\theta-1}{\theta}$.

Assuming optimal subsidy $1+\tau^S = \frac{\theta}{\theta-1}$:

$$\frac{C^H}{C} = 1 + \frac{\tau^D - \eta}{\lambda}\frac{1}{\theta-1}$$

$$\frac{C^S}{C} = 1 + \frac{\eta - \tau^D}{1-\lambda}\frac{1}{\theta-1}$$

Therefore, SS inequality $\Gamma \equiv \frac{C^S}{C^H}$ is modulated by 1. the SS net markup (without subsidy); 2. the profit redistribution every period τ^D and 3. the share of taxes levied on H to finance the subsidy. Crucially, we assume that this product is "small" so that the distortion is first-order; this amount to assuming that only a small fraction of the subsidy is paid for by the hand-to-mouth, or in any case something close to the share of profits they get indirectly through taxation τ^D.

Approximated welfare Given the assumption of uniform hours worked decided by the union, aggregate welfare per period will now be (see Ascari et al 2017 and Bilbiie Ragot 2020) $U(.) = \lambda U\left(C_t^H\right) + (1-\lambda) U\left(C_t^S\right) - \frac{\xi}{1+\varphi} N_t^{1+\varphi}$.

$$\begin{aligned}U_t - U &= \lambda\left(C^H\right)^{1-\sigma^{-1}}\left(c_t^H + \frac{1-\sigma^{-1}}{2}\left(c_t^H\right)^2\right) + (1-\lambda)\left(C^S\right)^{1-\sigma^{-1}}\left(\hat{c}_t^S + \frac{1-\sigma^{-1}}{2}\left(\hat{c}_t^S\right)^2\right) + \\ &\quad + U_N N\left(\hat{n}_t + \frac{1+\varphi}{2}\left(\hat{n}_t\right)^2\right)\end{aligned}$$

Because of the union assumption and under the subsidy $W=1$ we have $U_N = -U_C = -C^{-\sigma^{-1}}$.

$$\begin{aligned}U_t - U = &\left(C^S\right)^{-\sigma^{-1}}\left(\left(1-\lambda\right) C^S \hat{c}_t^S + \left(\frac{C^H}{C^S}\right)^{-\sigma^{-1}} \lambda C^H \hat{c}_t^H - \left(\frac{C}{C^S}\right)^{-\sigma^{-1}} N\hat{n}_t\right) \\ &+ (1-\lambda)\left(C^S\right)^{1-\sigma^{-1}}\frac{1-\sigma^{-1}}{2}\left(\hat{c}_t^S\right)^2 + \lambda\left(C^H\right)^{1-\sigma^{-1}}\left(\frac{1-\sigma^{-1}}{2}\left(\hat{c}_t^H\right)^2\right) - C^{1-\sigma^{-1}}\frac{1+\varphi}{2}\left(\hat{n}_t\right)^2\end{aligned}$$

The Economy Resource Constraint approximated to second order is:

$$\begin{aligned}
&\lambda C_t^H + (1-\lambda) C_t^S \\
&\simeq (1-\lambda) C^S \hat{c}_t^S + \lambda C^H \hat{c}_t^H + \frac{1}{2}\left(\lambda C^H \left(\hat{c}_t^H\right)^2 + (1-\lambda) C^S \left(\hat{c}_t^S\right)^2\right) \\
&= N_t \left(1 - \frac{\psi}{2}\pi_t^2\right) \\
&\simeq N\hat{n}_t + \frac{1}{2} N\hat{n}_t^2 - \frac{\psi\pi}{1-\frac{\psi}{2}\pi^2}\pi_t - \frac{1}{2}\frac{\psi}{1-\frac{\psi}{2}\pi^2}\pi_t^2 \\
&= N\hat{n}_t + \frac{1}{2} N\hat{n}_t^2 + \hat{\Delta}_t \\
\hat{\Delta}_t &= -\frac{\nu\pi}{1-\frac{\nu}{2}\pi^2}\pi_t - \frac{1}{2}\frac{\nu}{1-\frac{\nu}{2}\pi^2}\pi_t^2
\end{aligned}$$

Use in approximated welfare:

$$\begin{aligned}
U_t - U = \left(C^S\right)^{-\sigma^{-1}} &\left((1-\lambda) C^S \hat{c}_t^S + \left(\frac{C^H}{C^S}\right)^{-\sigma^{-1}} \lambda C^H \hat{c}_t^H - \left(\frac{C}{C^S}\right)^{-\sigma^{-1}} N\hat{n}_t\right) \\
&+ (1-\lambda)\left(C^S\right)^{1-\sigma^{-1}} \frac{1-\sigma^{-1}}{2}\left(\hat{c}_t^S\right)^2 + \lambda \left(C^H\right)^{1-\sigma^{-1}}\left(\frac{1-\sigma^{-1}}{2}\left(\hat{c}_t^H\right)^2\right) - C^{1-\sigma^{-1}}\frac{1+\varphi}{2}\left(\hat{n}_t\right)^2
\end{aligned}$$

$$\begin{aligned}
U_t - U &= \left(C^S\right)^{-\sigma^{-1}} \begin{pmatrix} (1-\lambda) C^S \hat{c}_t^S + \left(\frac{C^H}{C^S}\right)^{-\sigma^{-1}} \lambda C^H \hat{c}_t^H - \left(\frac{C}{C^S}\right)^{-\sigma^{-1}} N\hat{n}_t \\ +\lambda \left(\frac{C^H}{C^S}\right)^{-\sigma^{-1}} C^H \frac{1-\sigma^{-1}}{2}\left(\hat{c}_t^H\right)^2 + (1-\lambda) C^S \frac{1-\sigma^{-1}}{2}\left(\hat{c}_t^S\right)^2 \\ -\left(\frac{C}{C^S}\right)^{-\sigma^{-1}} N\frac{1+\varphi}{2}\left(\hat{n}_t\right)^2 \end{pmatrix} \\
&= \left(C^S\right)^{-\sigma^{-1}} \begin{pmatrix} (1-\lambda) C^S \hat{c}_t^S + \lambda C^H \hat{c}_t^H - N\hat{n}_t + \\ +\frac{1}{2}\left(\lambda C^H \left(\hat{c}_t^H\right)^2 + (1-\lambda) C^S \left(\hat{c}_t^S\right)^2\right) - \frac{1}{2} N\hat{n}_t^2 \\ +\lambda\left(\left(\frac{C^H}{C^S}\right)^{-\sigma^{-1}} - 1\right) C^H \hat{c}_t^H - \left(\left(\frac{C}{C^S}\right)^{-\sigma^{-1}} - 1\right) N\hat{n}_t \\ -(1-\lambda) C^S \frac{\sigma^{-1}}{2}\left(\hat{c}_t^S\right)^2 - \lambda C^H \left(\frac{C^H}{C^S}\right)^{-\sigma^{-1}} \frac{\sigma^{-1}}{2}\left(\hat{c}_t^H\right)^2 \\ -\left(\frac{C}{C^S}\right)^{-\sigma^{-1}} N\frac{\varphi}{2}\hat{n}_t^2 + \frac{1}{2}\lambda C^H \left(\left(\frac{C^H}{C^S}\right)^{-\sigma^{-1}} - 1\right)\left(\hat{c}_t^H\right)^2 - \frac{1}{2}\left(\left(\frac{C}{C^S}\right)^{-\sigma^{-1}} - 1\right) N\hat{n}_t^2 \end{pmatrix} \\
&= \left(C^S\right)^{-\sigma^{-1}} \begin{pmatrix} (1-\lambda) C^S \hat{c}_t^S + \lambda C^H \hat{c}_t^H - N\hat{n}_t + \\ +\frac{1}{2}\left(\lambda C^H \left(\hat{c}_t^H\right)^2 + (1-\lambda) C^S \left(\hat{c}_t^S\right)^2\right) - \frac{1}{2} N\hat{n}_t^2 \\ +\lambda\left(\left(\frac{C^H}{C^S}\right)^{-\sigma^{-1}} - 1\right) C^H \left(\hat{c}_t^H + \frac{1}{2}\left(\hat{c}_t^H\right)^2\right) - \left(\left(\frac{C}{C^S}\right)^{-\sigma^{-1}} - 1\right) N\left(\hat{n}_t + \frac{1}{2}\hat{n}_t^2\right) \\ -(1-\lambda) C^S \frac{\sigma^{-1}}{2}\left(\hat{c}_t^S\right)^2 - \lambda C^H \left(\frac{C^H}{C^S}\right)^{-\sigma^{-1}} \frac{\sigma^{-1}}{2}\left(\hat{c}_t^H\right)^2 - \left(\frac{C}{C^S}\right)^{-\sigma^{-1}} N\frac{\varphi}{2}\hat{n}_t^2 \end{pmatrix}
\end{aligned}$$

The first two lines are equal to the inflation resource cost Δ_t from the ERC approximation. Use inequality notation $\Gamma = \frac{C^H}{C^S}$ and denote $\tilde{\Gamma} \equiv \frac{C^S}{C} = \frac{\Gamma}{\lambda + (1-\lambda)\Gamma}$ (which trivially collapses to 1 when $\Gamma = 1$).

$$\hat{U} = \left(C^S\right)^{-\sigma^{-1}} \begin{pmatrix} \hat{\Delta}_t \\ +\lambda\left(\Gamma^{\sigma^{-1}} - 1\right) C^H \left(\hat{c}_t^H + \frac{1}{2}\left(\hat{c}_t^H\right)^2\right) - \left(\tilde{\Gamma}^{\sigma^{-1}} - 1\right) N\left(\hat{n}_t + \frac{1}{2}\hat{n}_t^2\right) \\ -(1-\lambda) C^S \frac{\sigma^{-1}}{2}\left(\hat{c}_t^S\right)^2 - \lambda C^H \Gamma^{\sigma^{-1}} \frac{\sigma^{-1}}{2}\left(\hat{c}_t^H\right)^2 - \tilde{\Gamma}^{\sigma^{-1}} N\frac{\varphi}{2}\hat{n}_t^2 \end{pmatrix}$$

Now, crucially, observe that since the steady state distortion is "small", $\Gamma^{\sigma^{-1}} - 1$ is of order 1 $O(\parallel \zeta \parallel)$, terms involving its product with a second-order term $O\left(\parallel \zeta \parallel^2\right)$ will be of order 3 $O\left(\parallel \zeta \parallel^3\right)$ and drop from the second-order approximation; this applies to the terms in the second line above. Therefore, the first-order term becomes (written as "loss" with minus in front):

$$-\lambda\left(\Gamma^{\sigma^{-1}}-1\right)\left[\frac{C^H}{C}\hat{c}_t^H-\hat{n}_t\right]$$

Let us redo the linear approximation under the assumption of a labor union.

$$\begin{aligned}\frac{C^H}{C}c_t^H &= w_t+n_t+\frac{\tau^D}{\lambda}d_t=\left(1-\frac{\tau^D}{\lambda}\right)w_t+n_t\\ &= \left(1-\frac{\tau^D}{\lambda}\right)w_t+c_t\end{aligned}$$

Labor union schedule and H budget constraint imply:

$$w_t=\varphi n_t+\sigma^{-1}c_t$$

$$\frac{C^H}{C}c_t^H=\left[1+\left(1-\frac{\tau^D}{\lambda}\right)\left(\varphi+\sigma^{-1}\right)\right]c_t$$

Let:

$$\chi_u=1+\left(1-\frac{\tau^D}{\lambda}\right)\left(\varphi+\sigma^{-1}\right)$$

be the key elasticity for the case of a union. We immediately obtain by substitution:

$$\begin{aligned}\frac{C^H}{C}c_t^H &= \chi_u c_t\\ (1-\lambda)\frac{C^S}{C}c_t^S &= (1-\lambda\chi_u)\,c_t\end{aligned}$$

Therefore, the linear term is:

$$-\lambda\left(\Gamma^{\sigma^{-1}}-1\right)(\chi_u-1)\,c_t$$

Denoting consumption inequality (as deviation/gap, in percentage of SS consumption) by:

$$\hat{\gamma}_t=\frac{C^H}{C}c_t^H-\frac{C^S}{C}c_t^S$$

it is clear that the linear term becomes:

$$\lambda(1-\lambda)\left(\Gamma^{\sigma^{-1}}-1\right)\hat{\gamma}_t,$$

illustrating that there is a first-order, linear benefit to reducing inequality. Evidently, if we reintroduced transfers that could be used over the cycle these would be used to perfectly offset this linear gap.

Next, let us focus on the *second-order terms* in the approximation. Divide throughout by C

$$-(1-\lambda)\frac{C^S}{C}\frac{\sigma^{-1}}{2}\left(\hat{c}_t^S\right)^2-\lambda\frac{C^H}{C}\Gamma^{\sigma^{-1}}\frac{\sigma^{-1}}{2}\left(\hat{c}_t^H\right)^2-\tilde{\Gamma}^{\sigma^{-1}}\frac{\varphi}{2}\hat{n}_t^2$$

Quadratic terms can be expressed as a function of aggregate consumption (output). Notice that in evaluating these quadratic terms we can use first-order approximations of the optimality conditions (higher order terms imply terms of order $O\left(\|\zeta\|^3\right)$).

Recall the first order expressions for individual consumptions above. To second order we thus have

$$\begin{aligned}\left(c_t^H\right)^2 &= \frac{\chi_u^2}{\left(\frac{C^H}{C}\right)^2}c_t^2+O\left(\|\zeta\|^3\right);\\ \left(c_t^S\right)^2 &= \left(\frac{1-\lambda\chi_u}{(1-\lambda)\frac{C^S}{C}}\right)^2c_t^2+O\left(\|\zeta\|^3\right);\end{aligned}$$

Replacing in the per-period welfare, considering only the "real" part (abstracting from inflation) for notational convenience and ignoring terms independent of policy and of order larger than 2, and finally writing as a "loss" (multiplying by -1):

$$(1-\lambda)\frac{C^S}{C}\frac{\sigma^{-1}}{2}\left(\hat{c}_t^S\right)^2+\lambda\frac{C^H}{C}\Gamma^{\sigma^{-1}}\frac{\sigma^{-1}}{2}\left(\hat{c}_t^H\right)^2+\tilde{\Gamma}^{\sigma^{-1}}\frac{\varphi}{2}\hat{n}_t^2$$

$$\frac{\sigma^{-1}}{2}\frac{C}{C^S}\left[\lambda\Gamma^{\sigma^{-1}+1}\chi_u^2+\frac{\left(1-\lambda\chi_u\right)^2}{1-\lambda}\right]c_t^2+\frac{\varphi}{2}\tilde{\Gamma}^{\sigma^{-1}}c_t^2$$

$$\frac{\sigma^{-1}}{2}\frac{C}{C^S}\left[\lambda\left(\Gamma^{\sigma^{-1}+1}-1\right)\chi_u^2+\left(1+\frac{\lambda}{1-\lambda}\left(\chi_u-1\right)^2\right)\right]c_t^2+\frac{\varphi}{2}\left[1+\left(\tilde{\Gamma}^{\sigma^{-1}}-1\right)\right]c_t^2$$

The last step in the argument is to notice that since we focus on the case of a small inequality distortion, with Γ close to 1 in that sense that $\left(\Gamma^{\sigma^{-1}}-1\right)$ is of order 1, terms involving its product with quadratic terms are third order $\left(O\left(\|\zeta\|^3\right)\right)$ and thus drop out from the quadratic approximation. Therefore, the quadratic part is:

$$\frac{\sigma^{-1}}{2}\left[1+\frac{\lambda}{1-\lambda}\left(\chi_u-1\right)^2\right]c_t^2+\frac{\varphi}{2}c_t^2$$

and it is identical to the case of an undistorted steady state.

The per-period loss function with a small inequality-distorted steady state is thus, up to a constant, as given in text:

$$\mathcal{L}=\frac{1}{2}\left\{\Lambda_c c_t^2+\pi_t^2\right\}-\Lambda c_t+t.i.p.+O\left(\|\zeta\|^3\right),$$

with

$$\Lambda_c=\frac{\sigma^{-1}+\varphi}{\psi}+\frac{\sigma^{-1}}{\psi}\frac{\lambda\left(\chi_u-1\right)^2}{1-\lambda};\Lambda\equiv\lambda\left(\Gamma^{\sigma^{-1}}-1\right)\frac{\chi_u-1}{\psi}.$$